Le centre commun INRIA-MSR

Jean-Jacques Lévy
INRIA

CENTRE DE RECHERCHE COMMUN

INRIA
MICROSOFT RESEARCH

MÉTHODES FORMELLES

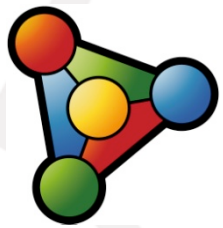
Composants mathématiques

Georges Gonthier
Benjamin Werner

Yves Bertot
Laurence Rideau
Laurent Théry

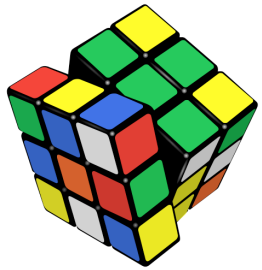
Preuves calculatoires

- assistant de preuves pour longues preuves de théorèmes mathématiques.
- réflexion des calculs dans la logique de Coq.



4 couleurs

Appel-Haken



groupes finis

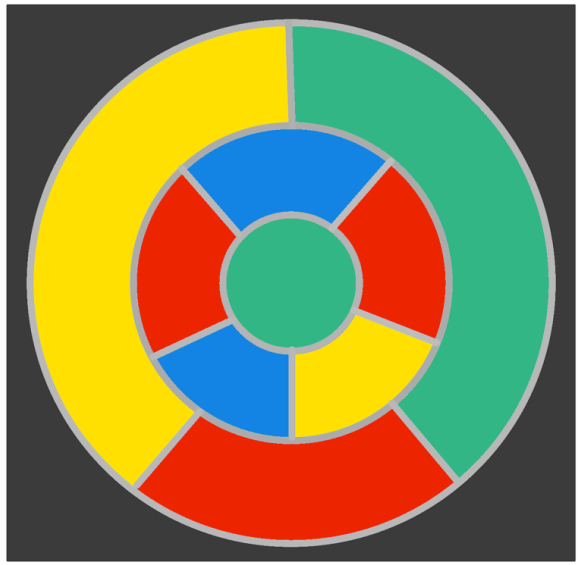
Feit-Thompson

Kepler

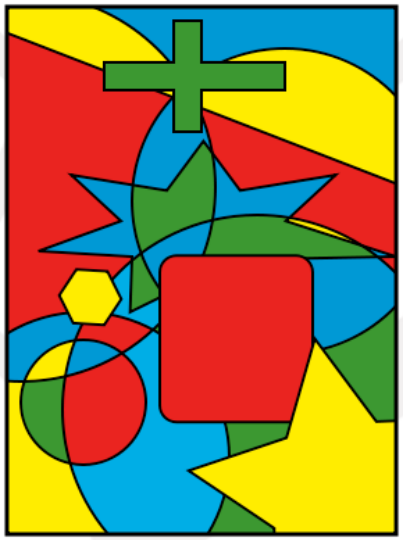
Hales

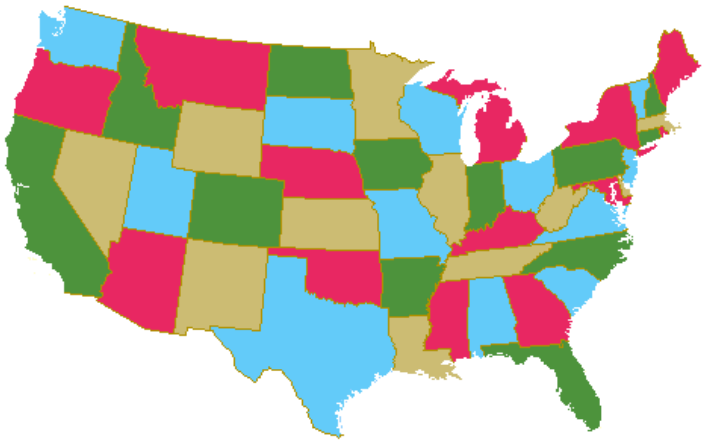
4 couleurs

???



bon coloriage





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```
Section R_props.

(* The ring axioms, and some useful basic corollaries. *)

Hypothesis mult1x : forall x, 1 * x = x.
Hypothesis mult0x : forall x : R, 0 * x = 0.
Hypothesis plus0x : forall x : R, 0 + x = x.
Hypothesis minusxx : forall x : R, x - x = 0.
Hypothesis plusA : forall x1 x2 x3 : R, x1 + (x2 + x3) = x1 + x2 + x3.
Hypothesis plusC : forall x1 x2 : R, x1 + x2 = x2 + x1.
Hypothesis multA : forall x1 x2 x3 : R, x1 * (x2 * x3) = x1 * x2 * x3.
Hypothesis multC : forall x1 x2 : R, x1 * x2 = x2 * x1.
Hypothesis distrR : forall x1 x2 x3 : R, (x1 + x2) * x3 = x1 * x3 + x2 * x3.

Lemma plusCA : forall x1 x2 x3 : R, x1 + (x2 + x3) = x2 + (x1 + x3).
Proof. move=> *; rewrite !plusA; congr (_ + _); exact: plusC. Qed.

Lemma multCA : forall x1 x2 x3 : R, x1 * (x2 * x3) = x2 * (x1 * x3).
Proof. move=> *; rewrite !multA; congr (_ * _); exact: multC. Qed.

Lemma distrL : forall x1 x2 x3 : R, x1 * (x2 + x3) = x1 * x2 + x1 * x3.
Proof. by move=> x1 x2 x3; rewrite !(multC x1) distrR. Qed.

Lemma oppK : involutive opp.
Proof.
by move=> x; rewrite -{2}[x]plus0x -(minusxx (- x)) plusC plusA minusxx plus0x.
Qed.

Lemma multm1x : forall x, -1 * x = -x.
Proof.
move=> x; rewrite -[_ * x]plus0x -(minusxx x) -{1}[x]mult1x plusC plusCA plusA.
by rewrite -distrR minusxx mult0x plus0x.
Qed.

Lemma mult_opp : forall x1 x2 : R, (- x1) * x2 = - (x1 * x2).
Proof. by move=> *; rewrite -multm1x -multA multm1x. Qed.

Lemma opp_plus : forall x1 x2 : R, - (x1 + x2) = - x1 - x2.
Proof.
by move=> x1 x2; rewrite -multm1x multC distrR -!(multC -1) !multm1x.
Qed.

Lemma RofSnE : forall n, RofSn n = n + 1.
Proof. by elim=> /= [|_ -> //]; rewrite plus0x. Qed.

Lemma Raddn : forall m n, (m + n)%N = m + n :> R.
Proof.
move=> m n; elim: m => /= [Im IHm]; first by rewrite plus0x.
by rewrite !RofSnE IHm plusC plusCA plusA.
Qed.

Lemma Rsubn : forall m n, m >= n -> (m - n)%N = m - n :> R.
```

-(DOS)-- **determinant.v** 42% (709,42) (Coq)

algèbre linéaire

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```
Lemma Rsubn : forall m n, m >= n -> (m - n)%N = m - n :> R.
Proof.
move=> m n; move/leq_add_sub=> Dm.
by rewrite -{2}Dm Raddn -plusA plusCA minusxx plusC plus0x.
Qed.

Lemma Rmuln : forall m n, (m * n)%N = m * n :> R.
Proof.
move=> m n; elim: m => /= [|m IHm]; first by rewrite mult0x.
by rewrite Raddn RofSnE IHm distrR mult1x plusC.
Qed.

Lemma RexpSnE : forall x n, RexpSn x n = x ^ n * x.
Proof. by move=> x; elim=> /= [|_ -> //]; rewrite mult1x. Qed.

Lemma mult_exp : forall x1 x2 n, (x1 * x2) ^ n = x1 ^ n * x2 ^ n.
Proof.
by move=> x1 x2; elim=> //= n IHn; rewrite !RexpSnE IHn -!multA (multCA x1).
Qed.

Lemma exp_addn : forall x n1 n2, x ^ (n1 + n2) = x ^ n1 * x ^ n2.
Proof.
move=> x n1 n2; elim: n1 => /= [|n1 IHn]; first by rewrite mult1x.
by rewrite !RexpSnE IHn multC multCA multA.
Qed.

Lemma Rexpn : forall m n, (m ^ n)%N = m ^ n :> R.
Proof. by move=> m; elim=> //= n IHn; rewrite Rmuln RexpSnE IHn multC. Qed.

Lemma exp0n : forall n, 0 < n -> 0 ^ n = 0.
Proof. by move=> [|[|n]] //= _; rewrite multC mult0x. Qed.

Lemma exp1n : forall n, 1 ^ n = 1.
Proof. by elim=> //= n IHn; rewrite RexpSnE IHn mult1x. Qed.

Lemma exp_muln : forall x n1 n2, x ^ (n1 * n2) = (x ^ n1) ^ n2.
Proof.
move=> x n1 n2; rewrite mulnC; elim: n2 => //= n2 IHn.
by rewrite !RexpSnE exp_addn IHn multC.
Qed.

Lemma sign_odd : forall n, (-1) ^ odd n = (-1) ^ n.
Proof.
move=> n; rewrite -{2}[n]odd_double_half addnC double_mul2 exp_addn exp_muln.
by rewrite /= multm1x oppK exp1n mult1x.
Qed.

Lemma sign_addb : forall b1 b2, (-1) ^ (b1 (+) b2) = (-1) ^ b1 * (-1) ^ b2.
Proof. by do 2!case; rewrite //= ?multm1x ?mult1x ?oppK. Qed.

Lemma sign_permM : forall d (s t : permType d),
-(DOS)--  determinant.v   45% (760,61)    (coq)
```

algèbre linéaire

```
rewrite isum0 ?plus0x // => i'; rewrite andbT; move/negbET->; exact: mult0x.
Qed.

Lemma matrix_transpose_mul : forall m n p (A : M_(m, n)) (B : M_(n, p)),
   \^t (A *m B) =m \^t B *m \^t A.
Proof. split=> k i; apply: eq_isumR => j _; exact: multC. Qed.

Lemma matrix_mult1x : forall m n (A : M_(m, n)), A *m \1m =m A.
Proof.
move=> m n A; apply: matrix_transpose_inj.
by rewrite matrix_transpose_mul matrix_transpose_unit matrix_mult1x.
Qed.

Lemma matrix_distrR : forall m n p (A1 A2 : M_(m, n)) (B : M_(n, p)),
  (A1 +m A2) *m B =m A1 *m B +m A2 *m B.
Proof.
move=> m n p A1 A2 B; split=> i k /=; rewrite -isum_plus.
by apply: eq_isumR => j _; rewrite -distrR.
Qed.

Lemma matrix_distrL : forall m n p (A : M_(m, n)) (B1 B2 : M_(n, p)),
  A *m (B1 +m B2) =m A *m B1 +m A *m B2.
Proof.
move=> m n p A B1 B2; apply: matrix_transpose_inj.
rewrite matrix_transpose_plus !matrix_transpose_mul.
by rewrite -matrix_distrR -matrix_transpose_plus.
Qed.

Lemma matrix_multA : forall m n p q
   (A : M_(m, n)) (B : M_(n, p)) (C : M_(p, q)),
  A *m (B *m C) =m A *m B *m C.
Proof.
move=> m n p q A B C; split=> i l /=.
transitivity (\sum_(k) (\sum_(j) (A i j * B j k * C k l))).
  rewrite exchange_isum; apply: eq_isumR => j _; rewrite isum_distrL.
  by apply: eq_isumR => k _; rewrite multA.
by apply: eq_isumR => j _; rewrite isum_distrR.
Qed.

Lemma perm_matrixM : forall n (s t : S_(n)),
  perm_matrix (s * t)%G =m perm_matrix s *m perm_matrix t.
Proof.
move=> n; split=> i j /=; rewrite (isumD1 (s i)) // set11 mult1x -permM.
rewrite isum0 => [|j']; first by rewrite plusC plus0x.
by rewrite andbT; move/negbET->; rewrite mult0x.
Qed.

Lemma matrix_trace_plus : forall n (A B : M_(n)), \tr (A +m B) = \tr A + \tr B.
Proof. by move=> n A B; rewrite -isum_plus. Qed.

Lemma matrix_trace_scale : forall n x (A : M_(n)), \tr (x *sm A) = x * \tr A.
Proof. by move=> *; rewrite isum_distrL. Qed.
-(DOS)--  determinant.v   77% (1190,48)   (coq)
```

algèbre linéaire

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```
(* And now, finally, the title feature. *)

Lemma determinant_multilinear : forall n (A B C : M_(n)) i0 b c,
    row i0 A =m b *sm row i0 B +m c *sm row i0 C ->
    row' i0 B =m row' i0 A -> row' i0 C =m row' i0 A ->
  \det A = b * \det B + c * \det C.
Proof.
move=> n A B C i0 b c ABC.
move/matrix_eq_rem_row=> BA; move/matrix_eq_rem_row=> CA.
rewrite !isum_distrL -isum_plus; apply: eq_isumR => s _.
rewrite -!(multCA (_ ^ s)) -distrL; congr (_ * _).
rewrite !(@iprodD1 _ i0 (setA _)) // (matrix_eq_row ABC) distrR !multA.
by congr (_ * _ + _ * _); apply: eq_iprodR => i;
   rewrite andbT => ?; rewrite ?BA ?CA.
Qed.

Lemma alternate_determinant : forall n (A : M_(n)) i1 i2,
  i1 != i2 -> A i1 =1 A i2 -> \det A = 0.
Proof.
move=> n A i1 i2 Di12 A12; pose r := I_(n).
pose t := transp i1 i2; pose tr s := (t * s)%G.
have trK : involutive tr by move=> s; rewrite /tr mulgA transp2 mul1g.
have Etr: forall s, odd_perm (tr s) = even_perm s.
  by move=> s; rewrite odd_permM odd_transp Di12.
rewrite /(\det _) (isumID (@even_perm r)) /=; set S1 := \sum_(in _) _.
rewrite -{2}(minusxx S1); congr (_ + _); rewrite {}/S1 -isum_opp.
rewrite (reindex_isum tr); last by exists tr.
symmetry; apply: eq_isum => [s | s seven]; first by rewrite negbK Etr.
rewrite -multm1x multA Etr seven (negbET seven) multm1x; congr (_ * _).
rewrite (reindex_iprod t); last by exists (t : _ -> _) => i _; exact: transpK.
apply: eq_iprodR => i _; rewrite permM /t.
by case: transpP => // ->; rewrite A12.
Qed.

Lemma determinant_transpose : forall n (A : M_(n)), \det (\^t A) = \det A.
Proof.
move=> n A; pose r := I_(n); pose ip p : permType r := p^-1.
rewrite /(\det _) (reindex_isum ip) /=; last first.
  by exists ip => s _; rewrite /ip invgK.
apply: eq_isumR => s _; rewrite odd_permV /= (reindex_iprod s).
  by congr (_ * _); apply: eq_iprodR => i _; rewrite permK.
by exists (s^-1 : _ -> _) => i _; rewrite ?permK ?permKv.
Qed.

Lemma determinant_perm : forall n s, \det (@perm_matrix n s) = (-1) ^ s.
Proof.
move=> n s; rewrite /(\det _) (isumD1 s) //.
rewrite iprod1 => [|i _]; last by rewrite /= set11.
rewrite isum0 => [|t Dst]; first by rewrite plusC plus0x multC mult1x.
case: (pickP (fun i => s i != t i)) => [i ist | Est].
  by rewrite (iprodD1 i) // multCA /= (negbET ist) mult0x.
move: Dst; rewrite andbT; case/eqP.
-(DOS)--  determinant.v   81% (1256,4)    (coq)
```

algèbre linéaire

```
Lemma determinant1 : forall n, \det (unit_matrix n) = 1.
Proof.
move=> n; have:= @determinant_perm n 1%G; rewrite odd_perm1 => /= <-.
apply: determinant_extensional; symmetry; exact: perm_matrix1.
Qed.

Lemma determinant_scale : forall n x (A : M_(n)),
  \det (x *sm A) = x ^ n * \det A.
Proof.
move=> n x A; rewrite isum_distrL; apply: eq_isumR => s _.
by rewrite multCA iprod_mult iprod_id card_ordinal.
Qed.

Lemma determinantM : forall n (A B : M_(n)), \det (A *m B) = \det A * \det B.
Proof.
move=> n A B; rewrite isum_distrR.
pose AB (f : F_(n)) (s : S_(n)) i := A i (f i) * B (f i) (s i).
transitivity (\sum_(f) \sum_(s : S_(n)) (-1) ^ s * \prod_(i) AB f s i).
  rewrite exchange_isum; apply: eq_isumR => s _.
  by rewrite -isum_distrL distr_iprodA_isumA.
rewrite (isumID (fun f => uniq (fval f))) plusC isum0 ?plus0x => /= [|f Uf].
  rewrite (reindex_isum (fun s => val (pval s))); last first.
    have s0 : S_(n) := 1%G; pose uf (f : F_(n)) := uniq (fval f).
    pose pf f := if insub uf f is Some s then Perm s else s0.
    exists pf => /= f Uf; rewrite /pf (insubT uf Uf) //; exact: eq_fun_of_perm.
  apply: eq_isum => [s|s _]; rewrite ?(valP (pval s)) // isum_distrL.
  rewrite (reindex_isum (mulg s)); last first.
    by exists (mulg s^-1) => t; rewrite ?mulKgv ?mulKg.
  apply: eq_isumR => t _; rewrite iprod_mult multA multCA multA multCA multA.
  rewrite -sign_permM; congr (_ * _); rewrite (reindex_iprod s^-1); last first.
    by exists (s : _ -> _) => i _; rewrite ?permK ?permKv.
  by apply: eq_iprodR => i _; rewrite permM permKv ?set11 // -{3}[i](permKv s).
transitivity (\det (\matrix_(i, j) B (f i) j) * \prod_(i) A i (f i)).
  rewrite multC isum_distrL; apply: eq_isumR=> s _.
  by rewrite multCA iprod_mult.
suffices [i1 [i2 Ef12 Di12]]: exists i1, exists2 i2, f i1 = f i2 & i1 != i2.
  by rewrite (alternate_determinant Di12) ?mult0x => //= j; rewrite Ef12.
pose ninj i1 i2 := (f i1 == f i2) && (i1 != i2).
case: (pickP (fun i1 => ~~ set0b (ninj i1))) => [i1| injf].
  by case/set0Pn=> i2; case/andP; move/eqP; exists i1; exists i2.
case/(perm_uniqP f): Uf => i1 i2; move/eqP=> Dfi12; apply/eqP.
by apply/idPn=> Di12; case/set0Pn: (injf i1); exists i2; apply/andP.
Qed.

(* And now, the Laplace formula. *)

Definition cofactor n (A : M_(n)) (i j : I_(n)) :=
   (-1) ^ (val i + val j) * \det (row' i (col' j A)).

(* Same bug as determinant
Add Morphism cofactor with
```

-(DOS)-- **determinant.v** 85% (1284,0) (coq)

algèbre linéaire

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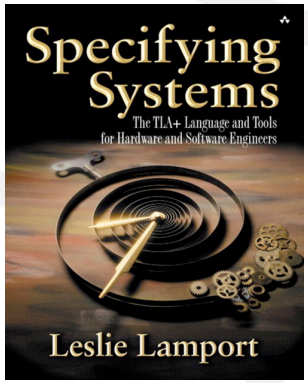
TLA+ et outils pour preuves

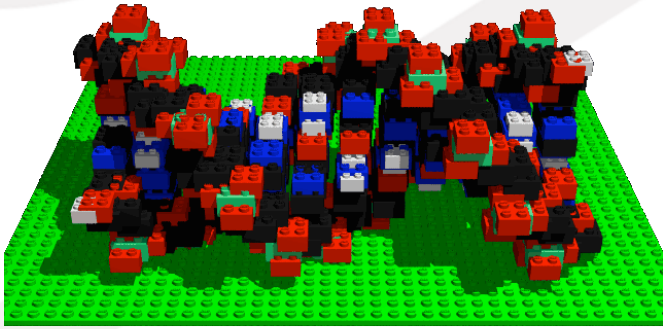
Damien Doligez
Leslie Lamport
Stephen Merz
Kaustuv Chaudury

Preuves naturelles

- théorie des ensembles en logique du premier ordre + temporalité.
- spécification et vérification des programmes concurrents.
- démonstration automatique.



TLA+



outils pour preuves formelles

Zenon

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Sécurité et preuves formelles

Cédric Fournet
Khartik Bhargavan
Ricardo Corin
James Leifer
Tamara Rezk
Gilles Barthe
J.-J. Lévy
Francesco Zappa Nardelli

Calcus distribués + Sécurité

- langages de programmation avec primitives de sécurité
- compilation certifiée de ces primitives en protocoles de bas niveau
- preuves formelles de protocoles probabilistes

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Sécurité et preuves formelles

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Calcus distribués + Sécurité

- langages de programmation avec primitives de sécurité
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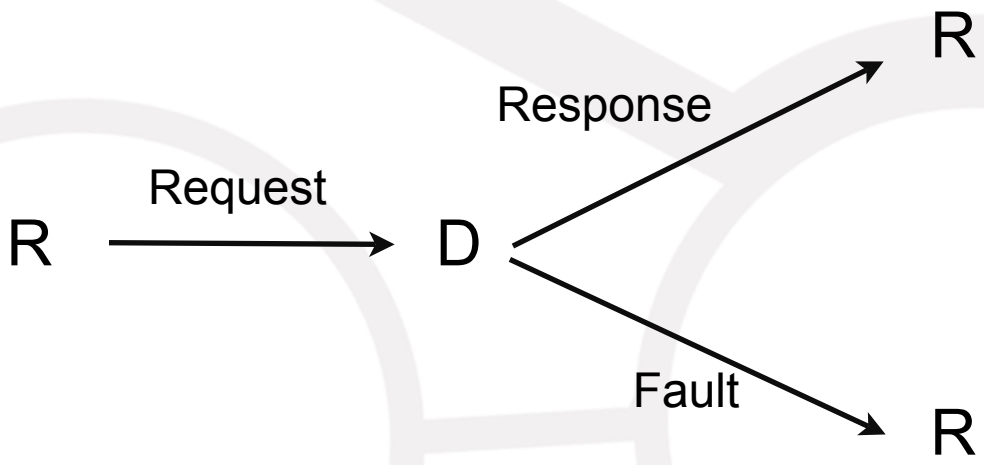
Echange simple



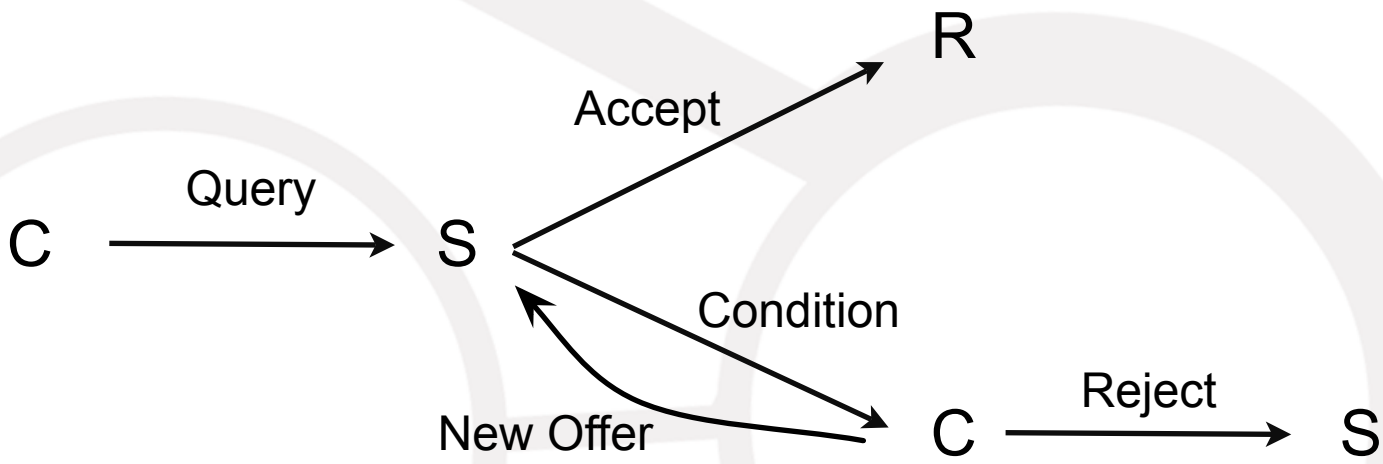

```
session S =
  role requester : int =
   !Request:string ;
   ?(Response:int + Fault:unit)

  role directory : string =
   ?Request:string;
   !(Response:int + Fault:unit)
```

Session declaration

```
let lookup name =
 S.requester ["client";"server"]
     (Request
          (name,
           {hResponse = (fun _ q → q) ;
            hFault = (fun _ x → failwith "Failed")
           }))
in lookup "Ricardo"
```

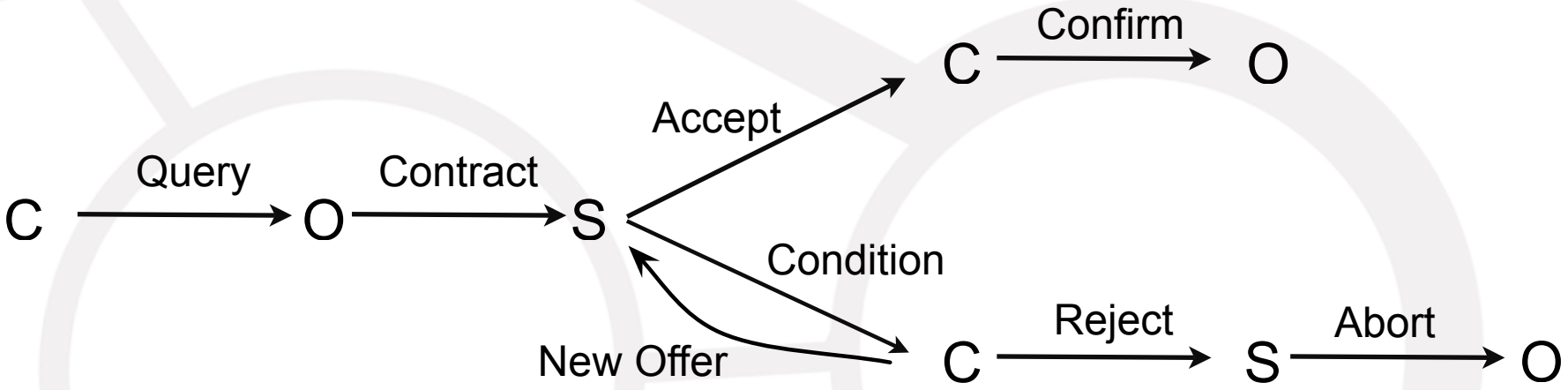
User code

Négociation



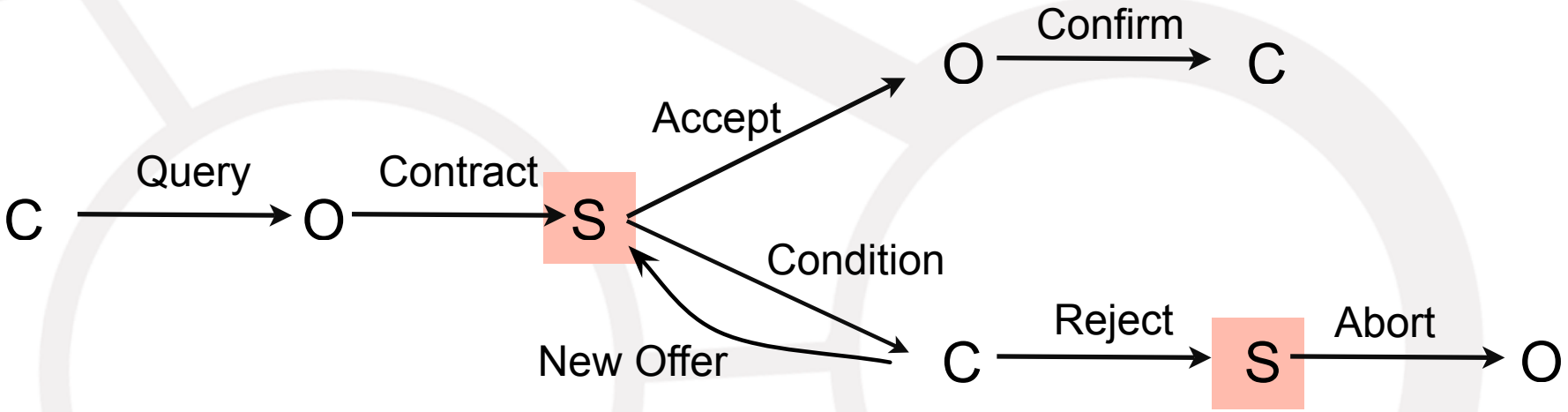

```
session S2 =
  role customer : string =
   !Query:int;
     mu start.?(Accept:unit +
                Condition:unit;!(NewOffer:int;start + Reject:unit))

   role store : string=
   ?Query:int;
     mu start.!(Accept:unit +
                Condition:unit;?(NewOffer:int;start + Reject:unit))
```

Trois partis



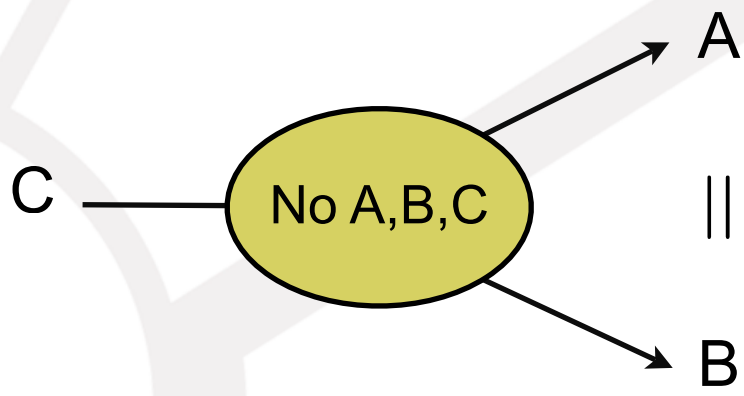

```
session S3 =
  role customer :string =
   !Query:int;
    mu start.?(Accept:unit;!Confirm:unit +
               Condition:unit; !(Newoffer:int;start + Reject:unit;))

  role store :string=
   ?Contract:int;
    mu start.!(Accept:unit +
               Condition:unit; ?(Newoffer:int;start + Reject:unit;!Abort:unit))

  role officer :string=
   ?Query:int;!Contract:int;?(Confirm:unit + Abort:unit)
```

Trois partis



- Some fork in protocols langages represent security threat

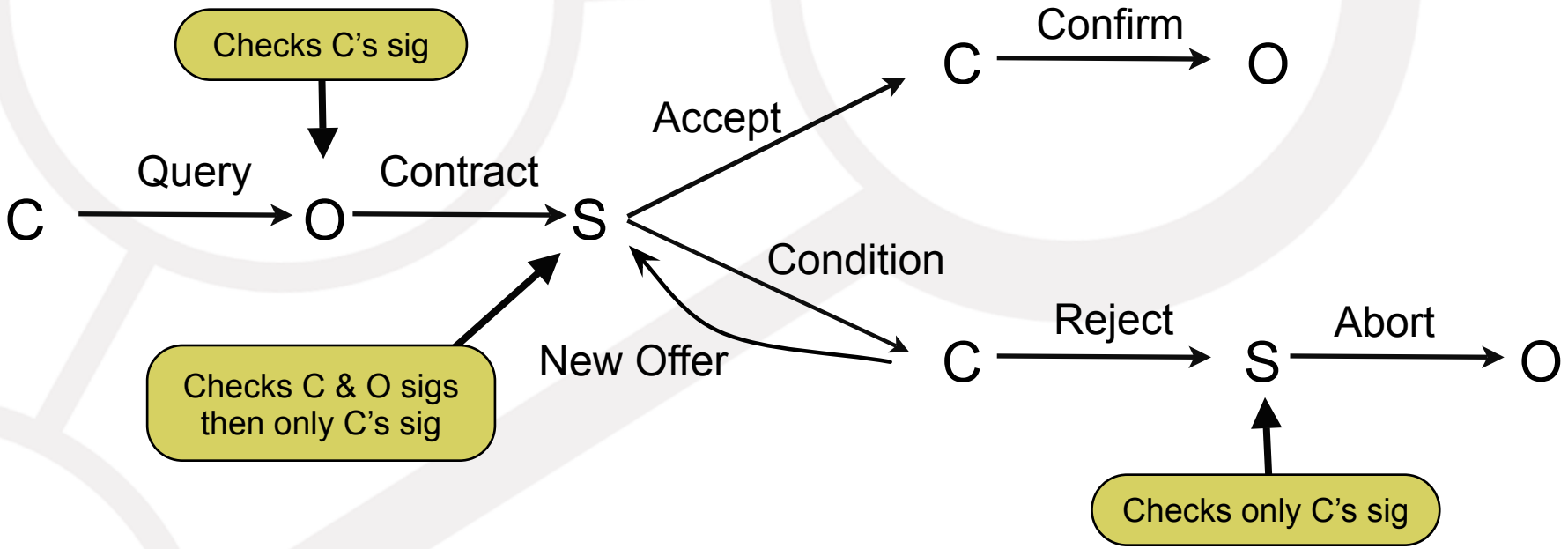
- Property



Role equality

Visibilité

- Minimal sequence of signatures that guarantees session compliance
- Example



INGÉNIÉRIE SCIENTIFIQUE

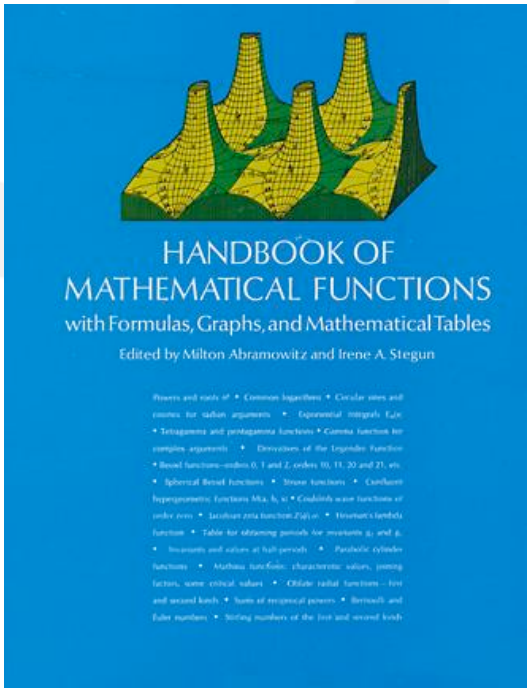
Encyclopédie dynamique des fonctions mathématiques

Bruno Salvy Alin Bostan Frédéric Chyzak

Calcul formel et Web pour les fonctions mathématiques utiles

- fonctions définies par équations différentielles linéaires.
- tables dynamiques de leur propriétés.
- génération de programmes pour les calculer.

Maple 11



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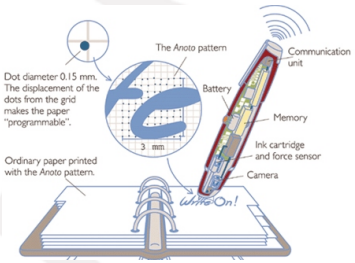
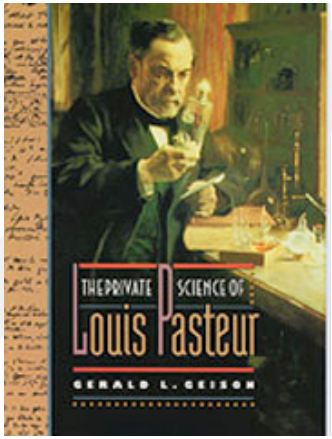
ReActivity

Jean-Daniel Fekete
Wendy Mackay
Mary Czerwinski
George Robertson

Gestion d'historiques d'expériences pour biologistes, historiens ou autres scientifiques:

- données à partir des cahiers de laboratoire et des ordinateurs.
- visualisation interactive de l'activité scientifique.
- aide à l'organisation du travail scientifique.





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INRIA MICROSOFT RESEARCH

Recherche et optimisation adaptative

Youssef Hamadi Marc Schoenauer

Solveur parallèle de contraintes et optimisation pour gros espaces de données:

- faciliter l'utilisation d'algorithmes combinatoires.
- automatiser le réglage fin des paramètres des solveurs.
- une solveur parallèle: ``disolver''.

MoGo

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Fouille d'images et vidéos pour applications scientifiques

Jean Ponce
Andrew Blake
Patrick Pérez
Cordelia Schmid

Vision par ordinateur et Apprentissage:

- environnement: détection de modifications dans les images satellitaires.
- archéologie et préservation de l'héritage culturel: modélisation 3D et reconnaissance dans les peintures ou photographies historiques.
- sociologie: modélisation de l'activité humaine et reconnaissance dans les archives vidéo.

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