

Lambda-Calculus

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Church's lambda-calculus is a kernel language for the design of programming languages and the study of their properties. The lambda-calculus is more directly connected to functional languages (Lisp, Scheme, SML, Ocaml, Haskell), but its type theory inspired many other languages (Java, C#, Scala). The lambda calculus has also many implications in mathematical logic. One of the most impressive results is the proof of the consistency of second-order arithmetic by Girard, through Howard correspondance. Another consequence is the formalization of mathematics (computer checked proof of the 4-colour theorem) or the verification of software and hardware in higher-order logics with computer proof-assistants (HOL, Isabelle, Coq).

This graduate course will follow and complete my two lectures at the 2nd Asian-Pacific Summer School on Formal Methods, August 23-24, Tsinghua. All classes will correspond to exercices. This course might motivate students to the theory of programming languages and to verification with formal methods.

Basic theorems	
Friday 09/03 10:30-12:00 FIT 1-small lecture hall	The calculus. Normal forms. Church-Rosser theorem (reminders). Residuals of redexes
Thursday 09/09 10:30-12:00 FIT 1-small lecture hall	Finite developments theorem. Parallel moves. Cube lemma. Residuals of reductions
Friday 09/10 10:30-12:00 FIT 1-small lecture hall	Strong normalisation vs Normalisation. Standardization theorem.
Semantics	
Tuesday 09/14 10:30-12:00 FIT 1-small lecture hall	Head normal forms. Bohm trees. Continuity theorem.
Thursday 09/16 10:30-12:00 FIT 1-small lecture hall	Observational equivalences. Bohm theorem. Extensionality
Friday 09/17 10:30-12:00 FIT 1-small lecture hall	Introduction to Scott's models.

Books on the lambda-calculus:

- Barendregt, Henk; The Lambda Calculus. Its Syntax and Semantics, 1984. Elsevier, 2nd edition, 1997.
- Barendregt, Henk; Lambda calculi with types, Handbook of logic in comp. science, Oxford, 1991.
- Barendregt, Henk; Dekkers, W. J. M.; Statman, Rick; Lambda calculus with types, Perspectives in Logic, Cambridge University Press, 2011.
- Church, Alonzo; The calculi of Lambda-Conversion, Princeton University Press, 1941.
- Curry, Haskell; Feys, R.; Combinatory logic. Volume 1. North Holland. 1958.
- Hindley, Roger; Seldin, Jonathan; Introduction to Combinators and Lambda-Calculus. Cambridge University Press. 1986.
- Girard, Jean-Yves; Taylor, Paul; Lafont, Yves; Proofs and Types. Cambridge University Press, 1989.
- Lévy, J.-J., École Polytechnique, 2005, <http://moscova.inria.fr/~levy/courses/X/M1/lambda>