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Inductive data types (I)

jean-jacques.levy@inria.fr

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Notes adapted from
Assia Mahboubi
(coq school 2010, Paris) and
Benjamin Pierce (software
foundations course, UPenn)

Plan

- easy proofs by simplification and reflexivity
- recursive types
- recursive definitions
- structural induction
- example1: lists
- example2: trees

Enumerated types

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Recap

- lambda notation for definition of functions
- Coq only allows typed lambda terms

Inductive declarations

An arbitrary type as assumed by:

```
Variable T : Type.
```

gives no a priori information on the nature, the number, or the properties of its inhabitants.

Inductive declarations

An inductive type declaration explains how the inhabitants of the type are built, by giving names to each construction rule:

Inductive declarations

Inductive types in *Coq* can be seen as the generalization of similar type constructions in more common programming languages.

They are in fact an extremely rich way of defining data-types, operators, connectives, specifications,...

They are at the core of powerful programming and reasoning techniques.

Enumeratives types (1/5)

Enumerated types are types which list and name exhaustively their inhabitants.

```
Inductive bool : Set := true : bool | false : bool.
```

Set is deprecated. Now use Type.

```
Inductive color : Type := black : color | white : color.
```

Enumeratives types (2/5)

Enumerated types are types which list and name exhaustively their inhabitants.

A new enumerated type:

```
Inductive day : Type :=
| monday | tuesday | wednesday |
| thursday | friday | saturday | sunday : day.
```

Enumeratives types (3/5)

Inspect the enumerated type inhabitants and assign values:

```
Definition negb (b : bool) :=
  match b with true => false | false => true end.
```

Enumeratives types (4/5)

```
Definition andb (b1:bool) (b2:bool) : bool :=
  match b1 with true => b2 | false => false  end.

Definition orb (b1:bool) (b2:bool) : bool :=
  match b1 with true => true | false => b2  end.
```

Enumeratives types (5/5)

Exercice Give definitions of predicates `work_day` and `weekend_day`.

Exercice Give definitions of predicates `black_if_workday` and white for weekends.