Reductions and Causality (IV)

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July 24, 2013

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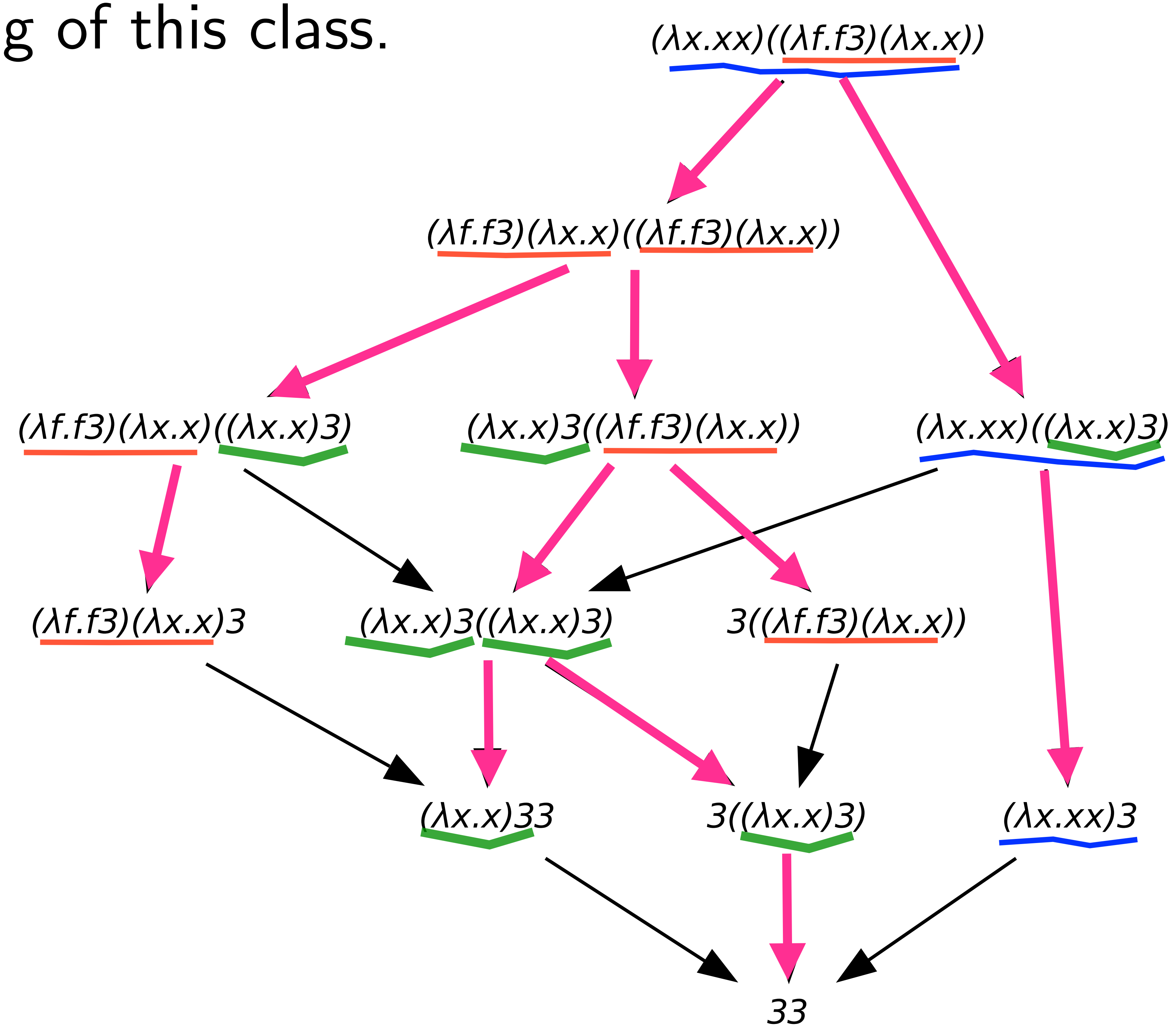
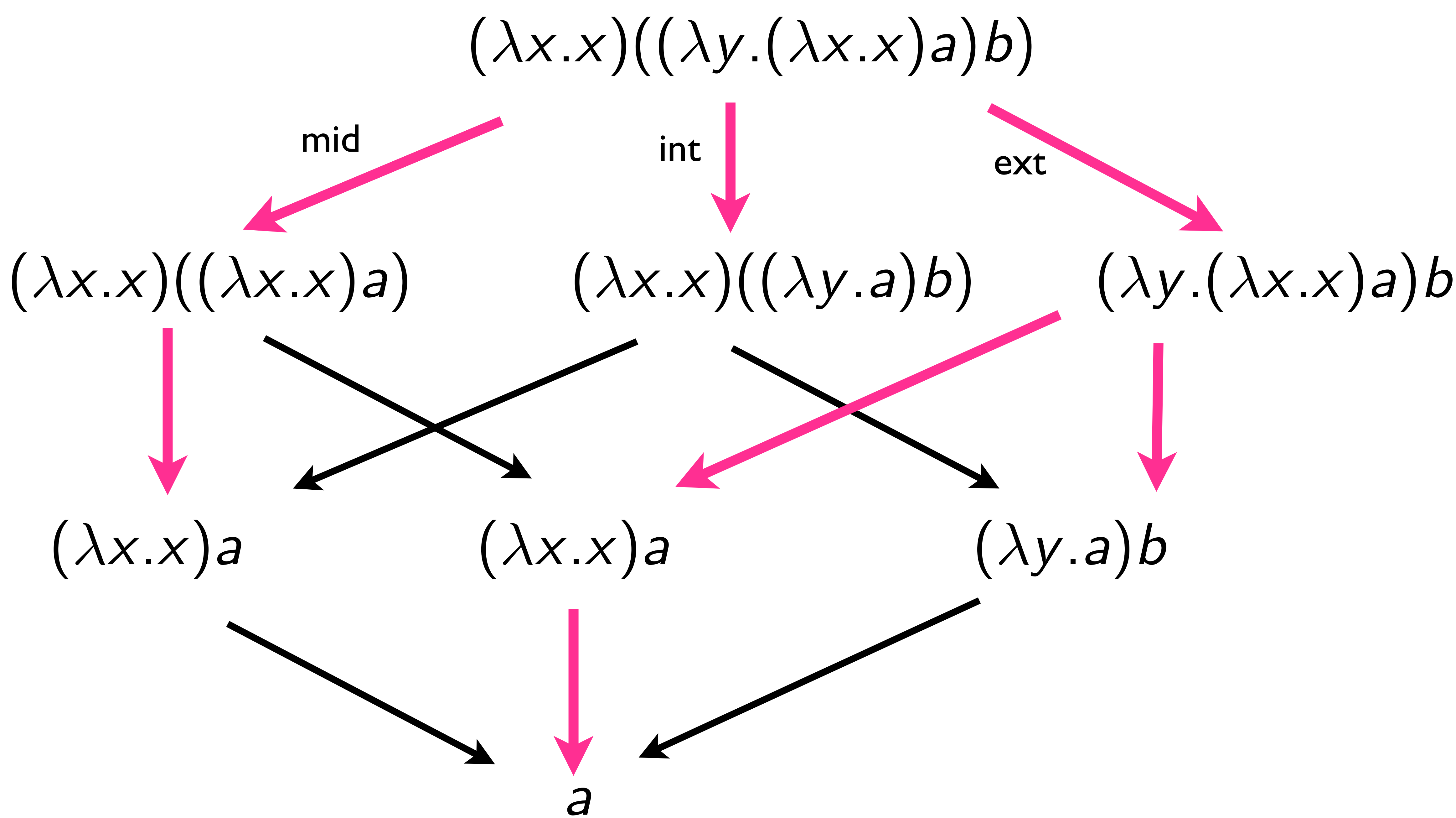
Exercices (1/4)

- Show all standard reductions in the 2 reduction graphs of beginning of this class.



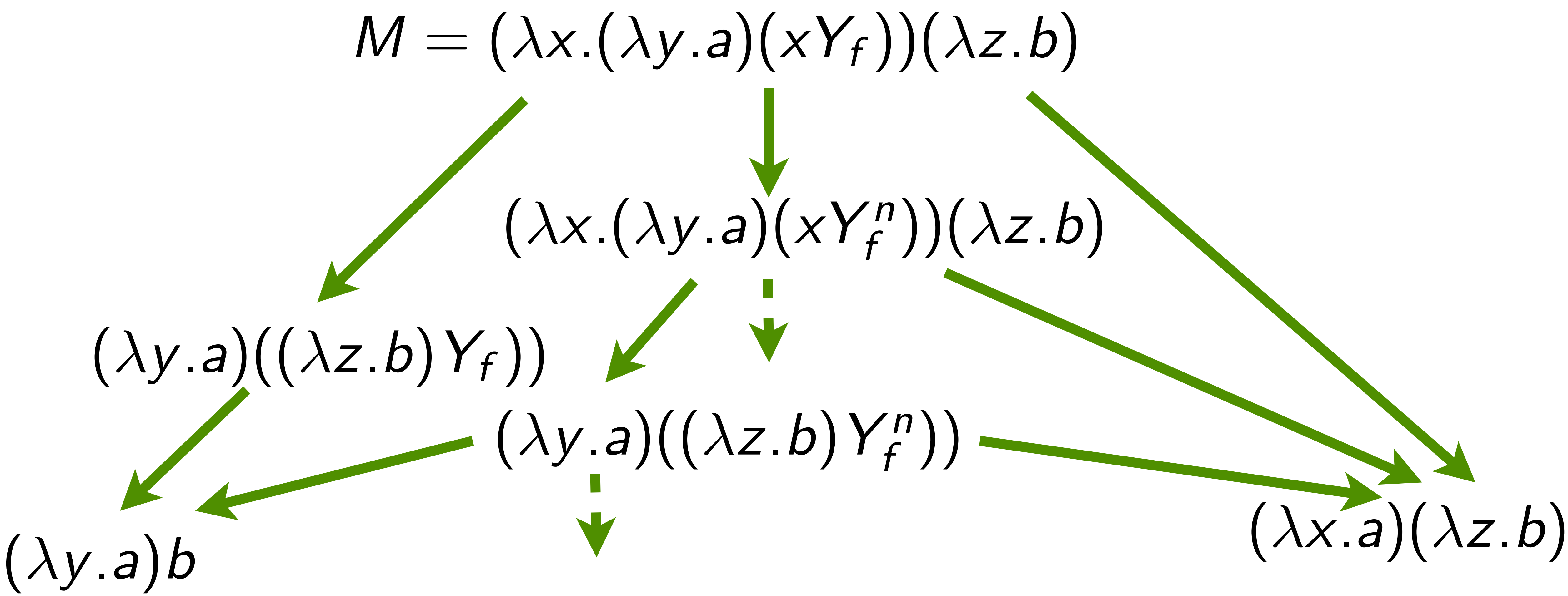
Exercices (2/4)

- Show all standard reductions in the 2 reduction graphs of beginning of this class.



Exercices (3/4)

- Find an example where there is no greatest lower bound of 2 reductions. (Hint: you should use K-terms)



$$Y_f = (\lambda x.f(xx))(\lambda x.f(xx))$$

Exercices (4/4)

- Show that there is inf-lattice of reductions in λI-calculus.

  $\rho_{\mathsf{st}} : M \xrightarrow{\star} N,\ \sigma_{\mathsf{st}} : M' \xrightarrow{\star} N,\ \tau : M \xrightarrow{\star} M'$

  then $|\rho_{\mathsf{st}}| \geq |\sigma_{\mathsf{st}}| + |\tau|$

Plan

- redexes and their history
- creation of redexes
- redex families
- finite developments
- finite developments+
- infinite reductions, strong normalization

Redex families

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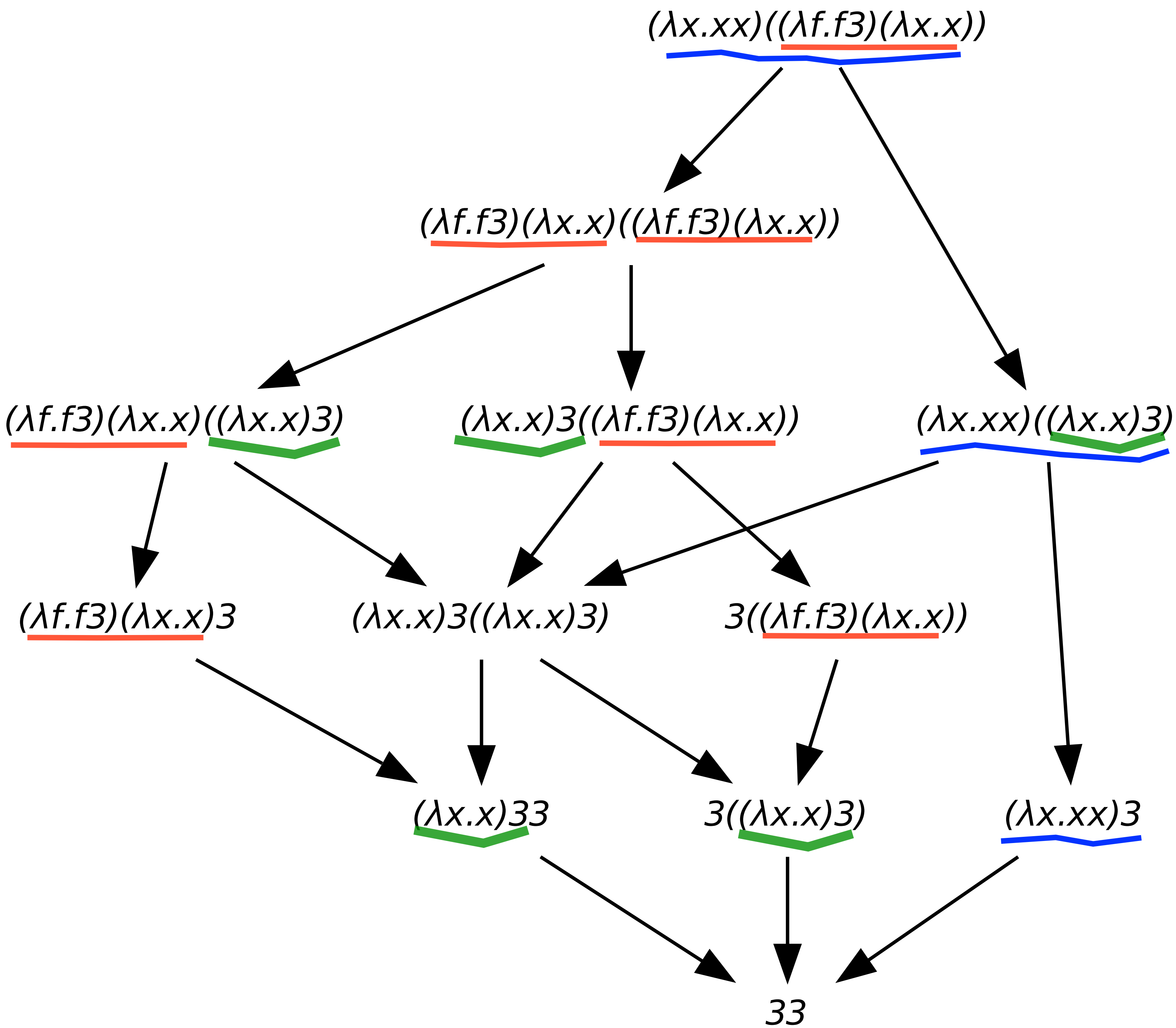
Initial redexes -- New redexes (1/3)



- **Red** and **blue** are initial redexes. **Green** is new.

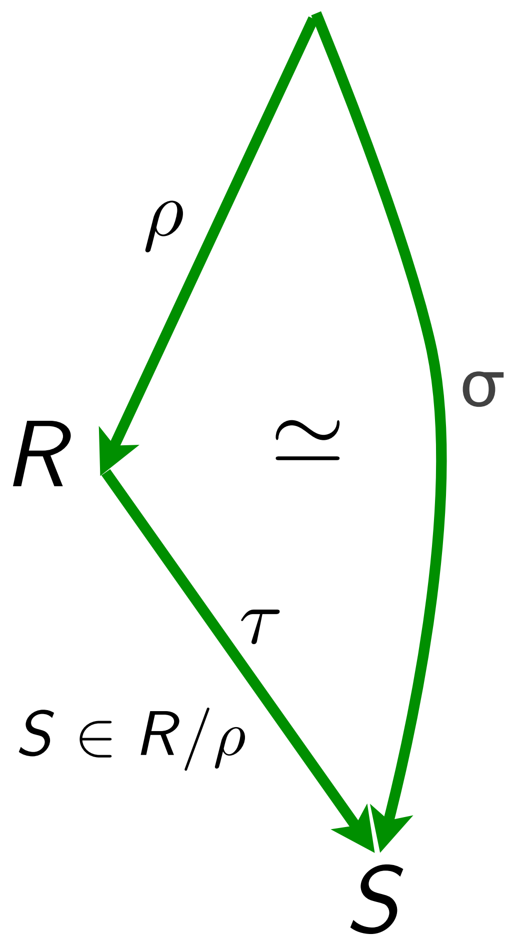
Redexes and their history (2/3)

- **Notation** [historical redexes]
  We write $\langle \rho, R \rangle$ when $\rho : M \xrightarrow{\star} N$ and R is redex in N.

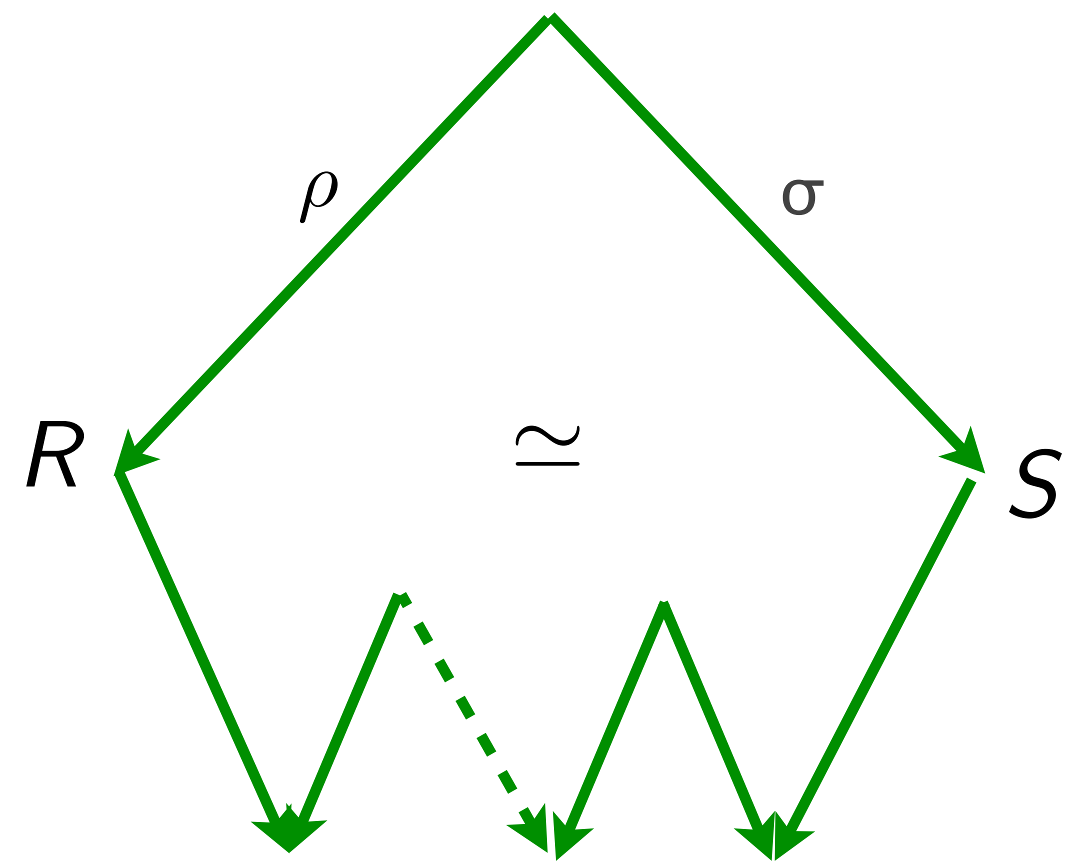
- **Definition** [copies of redexes]
  $\langle \rho, R \rangle \leq \langle \sigma, S \rangle$ when $\rho \sqsubseteq \sigma$ and $S \in R/(\sigma/\rho)$

- **Definition** [redex families]
  $\langle \rho, R \rangle \sim \langle \sigma, S \rangle$ stands for the symmetric and transitive closure of the copy relation.

Redexes and their history (3/3)

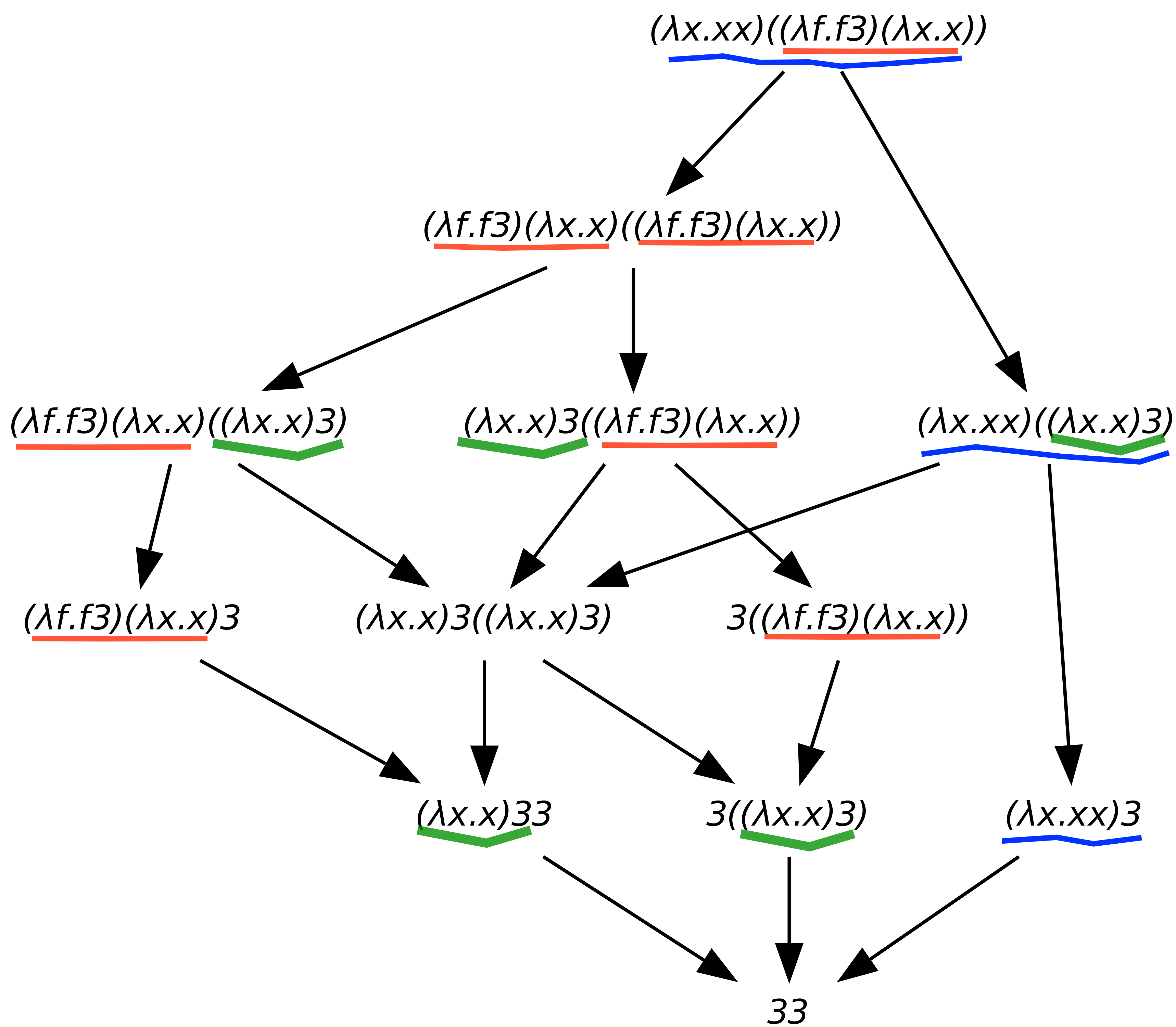


$\langle \rho, R \rangle \leq \langle \sigma, S \rangle$



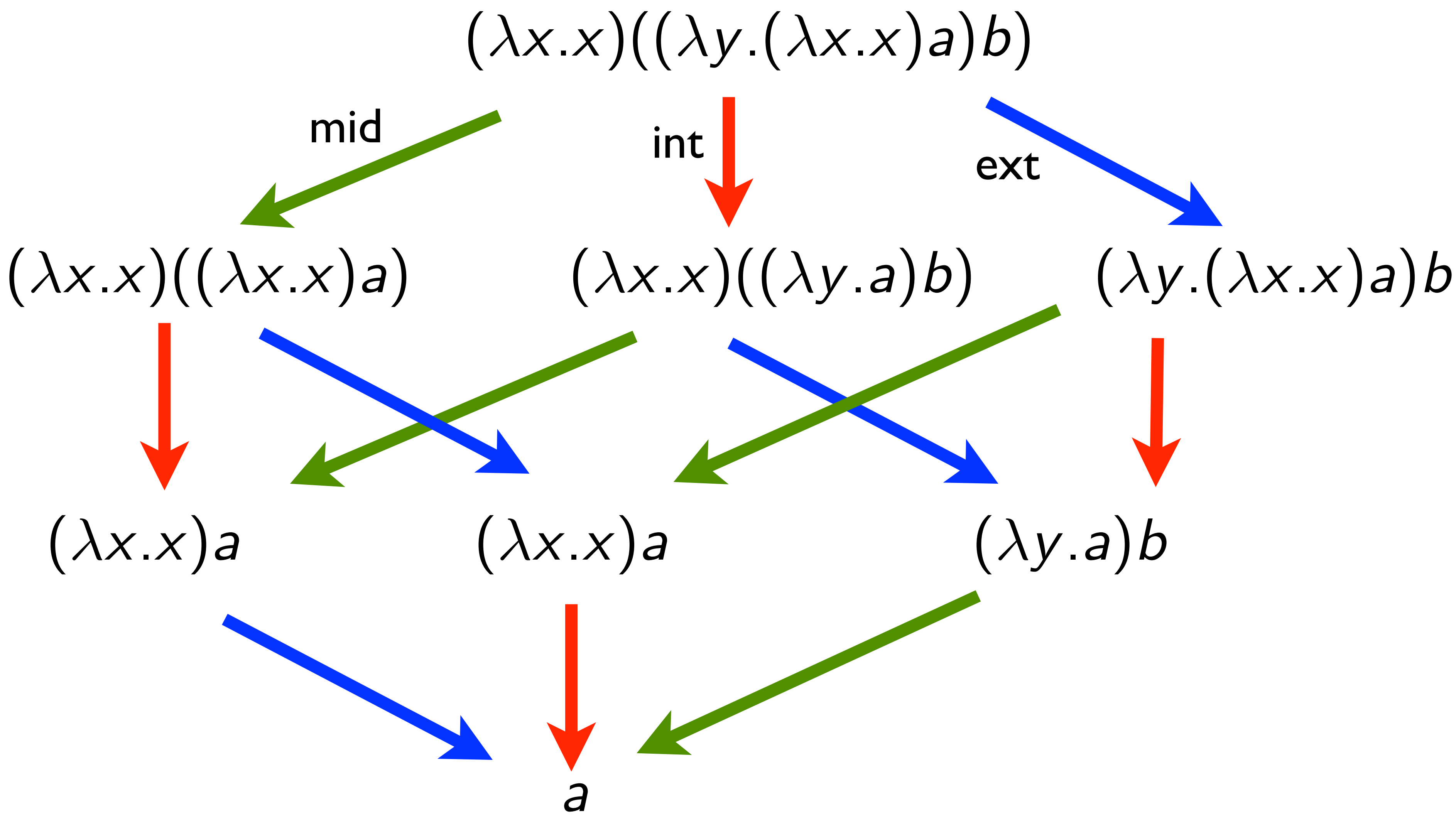
$\langle \rho, R \rangle \sim \langle \sigma, S \rangle$

Redex families (1/3)



- 3 redex families: **red**, **blue**, **green**.

Redex families (2/3)



- 3 redex families: **red**, **blue**, **green**.

Redex families (3/3)

- **Proposition**

(a) $T \in R/\rho, T \in S/\rho$ implies $R = S$

(b) $\rho \simeq \sigma$ implies $R/\rho = R/\sigma$

(c) $\langle \rho, R \rangle \leq \langle \tau, T \rangle, \langle \sigma, S \rangle \leq \langle \tau, T \rangle$ implies $\langle \rho, R \rangle \leq \langle \rho \sqcup \sigma, T' \rangle \leq \langle \tau, T \rangle,$
$\langle \sigma, S \rangle \leq \langle \rho \sqcup \sigma, T' \rangle \leq \langle \tau, T \rangle$

(d) $\langle \rho, R \rangle \leq \langle \tau, T \rangle, \langle \sigma, S \rangle \leq \langle \tau, T \rangle$ does not implies $\langle \tau_0, T_0 \rangle \leq \langle \rho, R \rangle,$
$\langle \tau_0, T_0 \rangle \leq \langle \sigma, S \rangle$ for some $\langle \tau_0, T_0 \rangle$

(e) $\langle \rho, R \rangle \sim \langle \sigma, S \rangle$ does not implies $\langle \tau_0, T_0 \rangle \leq \langle \rho, R \rangle, \langle \tau_0, T_0 \rangle \leq \langle \sigma, S \rangle$ for some
$\langle \tau_0, T_0 \rangle$

(f) $\langle \rho, R \rangle \sim \langle \sigma, S \rangle$ does not implies $\langle \rho, R \rangle \leq \langle \tau_0, T_0 \rangle, \langle \sigma, S \rangle \leq \langle \tau_0, T_0 \rangle$ for some
$\langle \tau_0, T_0 \rangle$

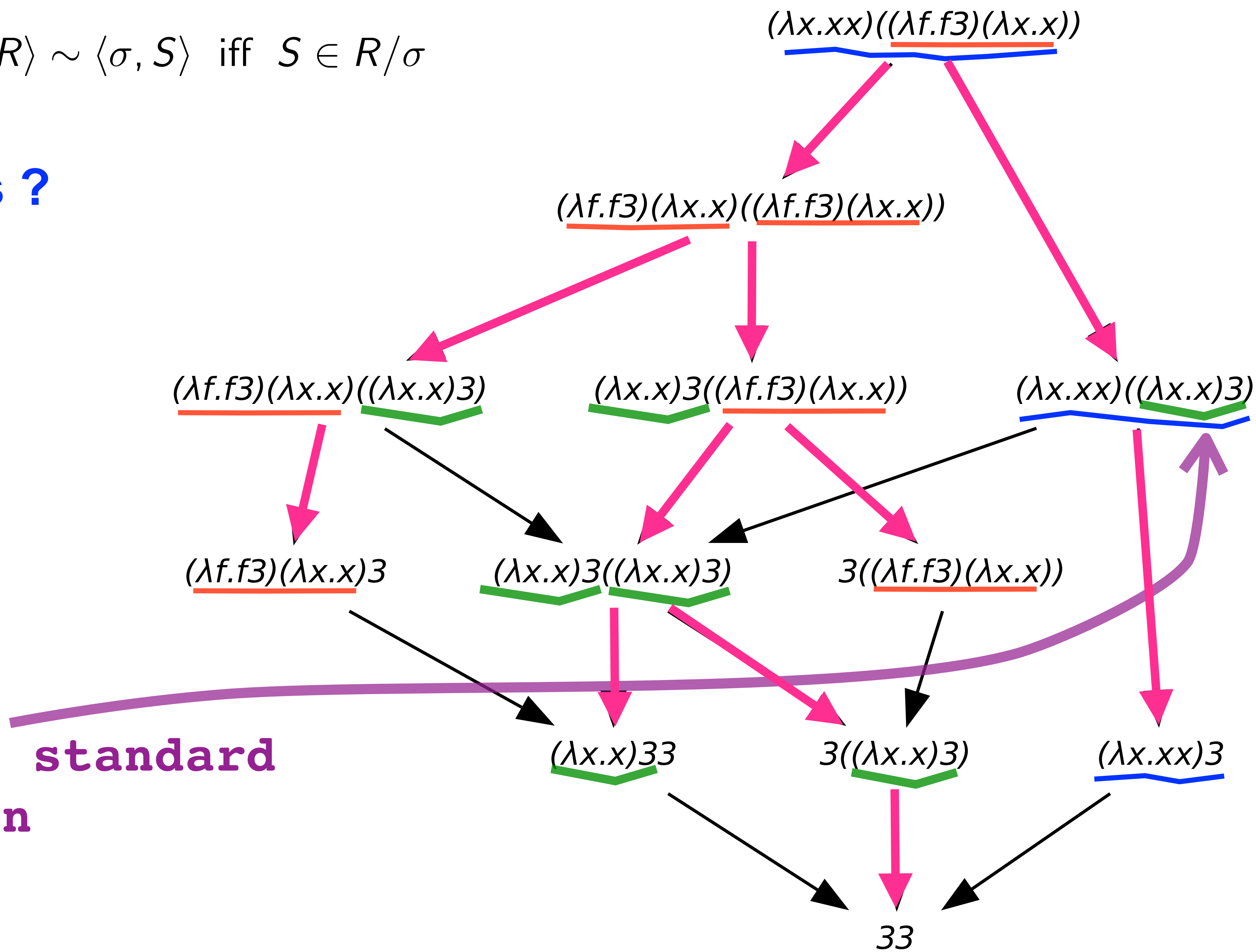
- **Question** Is there a canonical redex in each family ?

Canonical representatives (1/4)

- **Proposition** [initial redexes]

$$\langle 0, R\rangle \sim \langle \sigma, S\rangle \text{ iff } S \in R/\sigma$$

- **Others ?**



shortest standard reduction

Canonical representatives (2/4)

- **Definition** [extraction of canonical redex]

Let $M = (\lambda x.P)Q\, M_1 M_2 \cdots M_n$ and $\langle \rho_{\mathrm{st}}, R \rangle$ be historical redex from M and H is head redex in M.

$$\mathrm{extract}(H; \rho_{\mathrm{st}}, R) = H; \mathrm{extract}(\rho_{\mathrm{st}}, R)$$