MPRI Concurrency (course number 2-3) 2004-2005: π -calculus 9 December 2004

http://pauillac.inria.fr/~leifer/teaching/mpri-concurrency-2004/

James J. Leifer INRIA Rocquencourt

James.Leifer@inria.fr

9 December 2004

Today's plan

- exercises from last week
- review: barbed bisimilarity
- two natural congruences
- a family portrait

• weak barbed congruence and weak labelled bisimilarity correspond

Weak barbed bisimulation

Recall that a process P has a strong barb x, written $P \downarrow x$ iff there exists P_0 , P_1 , and \vec{y} such that $P \equiv \nu \vec{y}.(\overline{x}u.P_0 \mid P_1)$ and $x \notin \vec{y}.$

A process *P* has a weak barb *x*, written $P \Downarrow x$ iff there exists *P'* such that $P \longrightarrow^* P'$ and $P' \downarrow x$.

A relation \mathcal{R} is a weak barbed bisimulation if it is symmetric and for all $(P,Q) \in \mathcal{R}$

• if $P \longrightarrow P'$, there exists Q' such that $Q \longrightarrow^* Q'$ and $(P', Q') \in \mathcal{R}$;

• if $P \downarrow x$ then $Q \Downarrow x$.

Weak barbed bisimilarity, written $\dot{\approx}$, is the largest such relation.

Two possible equivalences (non-input congruences)

We write "equivalence" for "non input-prefixing congruence". Clearly \approx isn't an equivalence: $\overline{xy} \approx \overline{xz}$ but $- | x(u).\overline{u}w$ can distinguish them. There are two ways of building an equivalence:

9 December 2004

- Close up at the end: weak barbed equivalence, $\dot{\approx}^{\circ}$, is the largest equivalence included in $\dot{\approx}$. Concretely, $P \approx^{\circ} Q$ iff for all contexts $C \in \mathcal{E}$ we have $C[P] \approx C[Q]$. Check!
- Close up at every step: weak barbed reduction equivalence, \approx , is the largest relation \mathcal{R} such that \mathcal{R} is a weak barbed bisimulation and an equivalence. Concretely, \approx is the largest symmetric relation \mathcal{R} such that for all $(P, Q) \in \mathcal{R}$,

- if $P \longrightarrow P'$, there exists Q' such that $Q \longrightarrow^* Q'$ and $(P', Q') \in \mathcal{R}$;

- if $P \downarrow x$ then $Q \Downarrow x$;

- for all
$$C \in \mathcal{E}$$
, we have $(C[P], C[Q]) \in \mathcal{R}$.

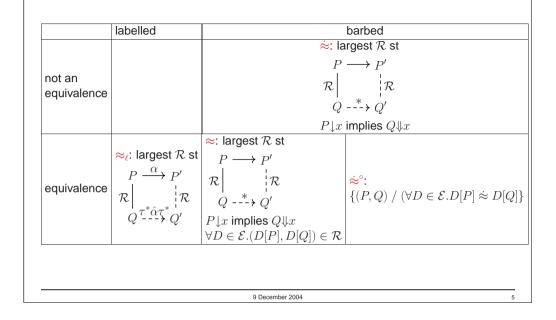
Check!

An extended family portrait

	strong	
	labelled	barbed
not an equivalence		"bisimilarity" $\dot{\sim}$
equivalence	"bisimilarity" \sim_ℓ	"equivalence" $\dot{\sim}^{\circ}$
		"reduction equivalence" \sim
congruence	"full bisimilarity" \simeq_ℓ	"congruence" \simeq°
		"reduction congruence" \simeq
	weak	
	labelled	barbed
not an equivalence		"bisimilarity" \dot{pprox}
equivalence	"bisimilarity" $pprox_\ell$	"equivalence" $\stackrel{.}{pprox}^{\circ}$
		"reduction equivalence" $pprox$
congruence	"full bisimilarity" \cong_{ℓ}	"congruence" $\stackrel{.}{\cong}^{\circ}$
		"reduction congruence" \cong

A detailed family portrait

4



What's the difference between pprox and \dot{pprox}° ?

- $\approx \subseteq \dot{\approx}^{\circ}$: Yes, trivially.
- $\approx \supseteq \approx^{\circ}$: Not necessarily.

Two difficult results due to Cédric Fournet and Georges Gonthier. "A hierarchy of equivalences for asynchronous cacluli". ICALP 1998. Journal version:

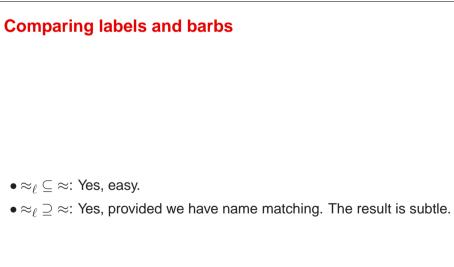
 $http://research.microsoft.com/{\sim} fournet/papers/a-hierarchy-of-equivalences-for-asynchronous-calculi.pdf$

– In general they're not the same. $\dot{\approx}^\circ$ is not even guaranteed to be a weak barbed bisimulation:

 $\begin{array}{ccc} P & C[P] \longrightarrow P' \\ \dot{\approx}^{\circ} \middle| & \dot{\approx} \middle| & & \dot{|} \\ Q & C[Q] \xrightarrow{*} Q' \end{array}$

– But for π -calculus, they coincide.

9 December 2004



9 December 2004

7

Name matching

Motivation: Which context can detect that $P \xrightarrow{\overline{xy}} P'$? It's easy to tell P can output on x; we just check $P \downarrow x$. If we want to test that this transition leads to P', we can take the context $C = - |x(u).k| \overline{k}$ for k fresh. Now

$$C[P] \longrightarrow P'$$

where $P' \not \mid k$.

But how do we detect that the message is y? We could try

$$C = - \mid x(u).(\overline{u} \mid y.k) \mid \overline{k}$$

but this risks having the \overline{u} and the *y* interact with the process in the hole.

Thus, we introduce a simple new construct called name matching:

$$P ::= \dots \\ [x = y].P$$

Reductions: $[x = x].P \longrightarrow P$

Labelled transitions: $[x = x] \cdot P \xrightarrow{\tau} P$

9 December 2004

case $\alpha = \overline{x}y$: Let $C = - | x(u).[u = y].k | \overline{k}$, where k is fresh. Then $C[P] \longrightarrow \longrightarrow P'$. Therefore, there exists Q such that $C[Q] \longrightarrow Q'$ and $P' \approx Q'$. Since $P' \not \downarrow k$, we have $Q' \not \downarrow k$, therefore $Q \xrightarrow{\tau} * \xrightarrow{\overline{x}y} \xrightarrow{\tau} * Q'$, as desired.

case $\alpha = \overline{x}(y)$ and $y \notin fn(Q)$: Let

$$C = - |x(u).(\overline{z}u | k | \prod_{w \in \mathsf{fn}(P)} [u = w].\overline{k}) | \overline{k}$$

where k and z are fresh. Then $C[P] \longrightarrow H_{z,y}[P']$ where

 $H_{z,y} = \boldsymbol{\nu} y.(\overline{z}y \mid -)$ Therefore, there exists Q'' such that $C[Q] \longrightarrow^* Q''$ and $H_{z,y}[P'] \approx Q''.$ Since $H_{z,y}[P'] \not \downarrow k$, we have $Q'' \not \downarrow k$. Thus there exists Q' such that $Q'' \equiv C'[Q']$ and $Q \xrightarrow{\tau} * \stackrel{\overline{x}(y)}{\longrightarrow} \stackrel{\tau}{\longrightarrow} * Q'$. Do we know $P' \approx Q'$?

Barbed equivalence is a weak labelled bisimulation

Theorem: $\approx_{\ell} \supseteq \approx$.

Proof: Consider $P \approx Q$ and suppose $P \xrightarrow{\alpha} P'$. (For simplicity, ignore structural congruence.)

case $\alpha = \tau$: Then $P \longrightarrow P'$. By definition, there exists Q' such that $Q \longrightarrow^* Q'$ and $P' \approx Q'$. Thus $Q \xrightarrow{\tau} Q'$ as desired.

case $\alpha = xy$: Let $C = - | \overline{xy}.k | \overline{k}$, where k is fresh. Then $C[P] \longrightarrow P'$. Therefore, there exists Q such that $C[Q] \longrightarrow^* Q'$ and $P' \approx Q'$. Since $P' \not \downarrow k$, we have $Q' \not \downarrow k$, therefore $Q \xrightarrow{\tau} * \xrightarrow{xy} \xrightarrow{\tau} * Q'$, as desired.

9 December 2004

Exercises for next lecture

1. Since the last lecture, the proof has been fixed by using #k everywhere. Prove from the definition of \approx that for $P \approx Q$ if $P \Downarrow x$ then $Q \Downarrow x$, and thus the contrapositive: if Q # x then P # x. 2. The last case of the proof relies on the following lemma: $H_{z,y}[P] \approx H_{z,y}[Q]$ implies $P \approx Q$, where $z \notin \operatorname{fn}(P) \cup \operatorname{fn}(Q)$. In the updated version of the proof you will find the definition $H_{z,y} = \nu y . (\overline{z}y \mid -)$. Hints...

In order to prove this, consider

 $\mathcal{R} = \{(P,Q) \mid z \notin \mathsf{fn}(P) \cup \mathsf{fn}(Q) \text{ and } H_{z,y}[P] \approx H_{z,y}[Q]\}.$

Our goal (as usual) is to prove that \mathcal{R} satisfies the same properties as \approx , and thus deduce that $\mathcal{R} \subseteq \approx$. Assume $(P, Q) \in \mathcal{R}$.

- \mathcal{R} is a bisimulation: Show that $P \longrightarrow P'$ implies that there exists Q' such that $Q \longrightarrow^* Q'$ and $(P', Q') \in \mathcal{R}$.
- \mathcal{R} preserves barbs: Show that $P \downarrow w$ implies $Q \Downarrow w$.
- \mathcal{R} is an equivalence: It is sufficient to show that $(C[P], C[Q]) \in \mathcal{R}$ where $C = \nu \vec{w}.(- | S)$. Hint: try to find a context C' such that $H_{z,y}[C[P]] \approx C'[H_{z,y}[P]]$ and the same for Q (perhaps using a labelled bisimilarity since we know $\approx_{\ell} \subseteq \approx$). You may have to distinguish between the cases $y \in \vec{w}$ and $y \notin \vec{w}$.

9 December 2004

12

