1. Design
   - *i*ML^F: an implicity-typed extension of System F
   - Types explained
   - *e*ML^F: an explicitly-typed version of *i*ML^F

2. Uses and Implementation
   - Examples
   - Type inference
   - Restrictions and extensions

ML^F for Everyone

(Users, Implementers, and Designers)

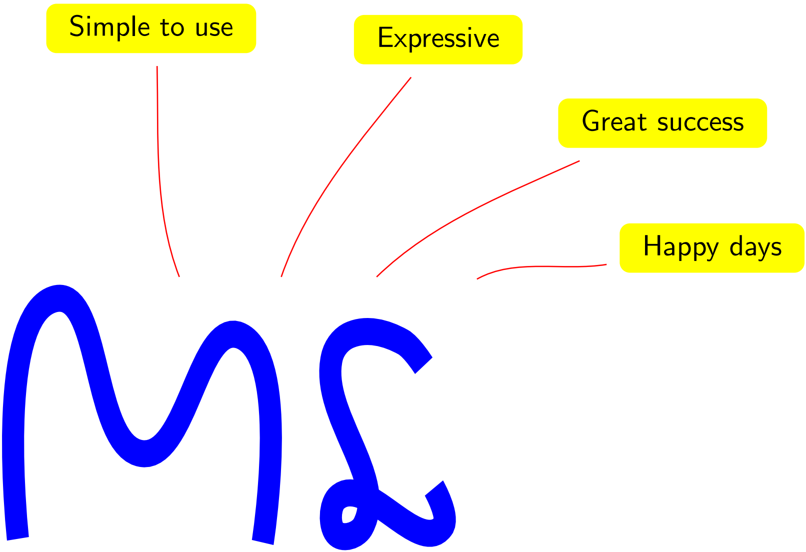
Didier Rémy

INRIA-Rocquencourt

ML Workshop

(Based on joint work with Didier Le Botlan and Boris Yakobowski)

Ms


Simple to use
Expressive
Great success
Happy days

F

ML

F

Ms

Expressive

Simple ~~extension~~ of ML
Simplification

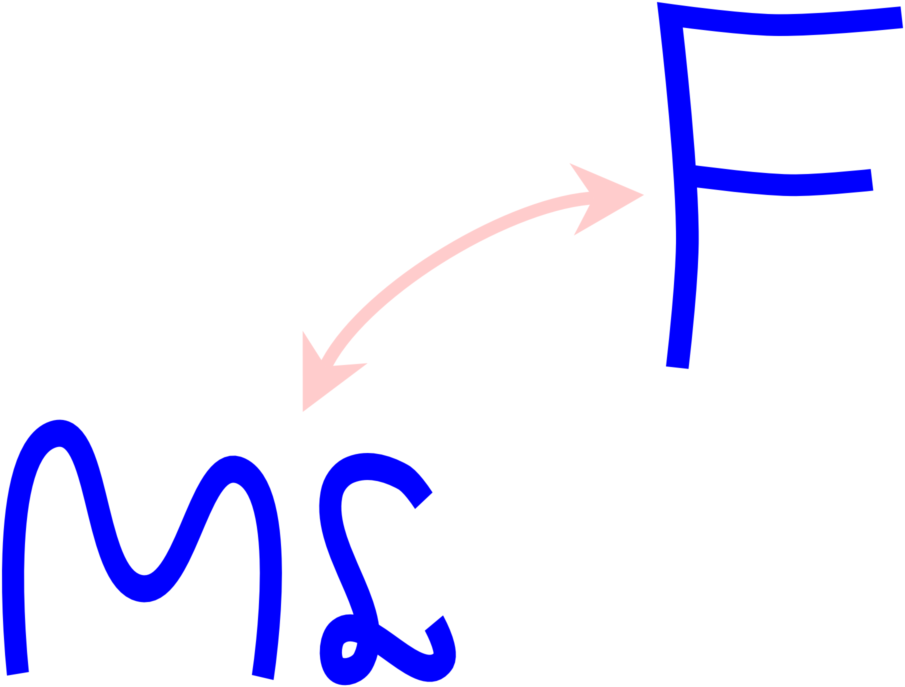
Even used in full scale languages such as Scala.

Full type inference is undecidable

F

Full type annotations are obfuscating

ML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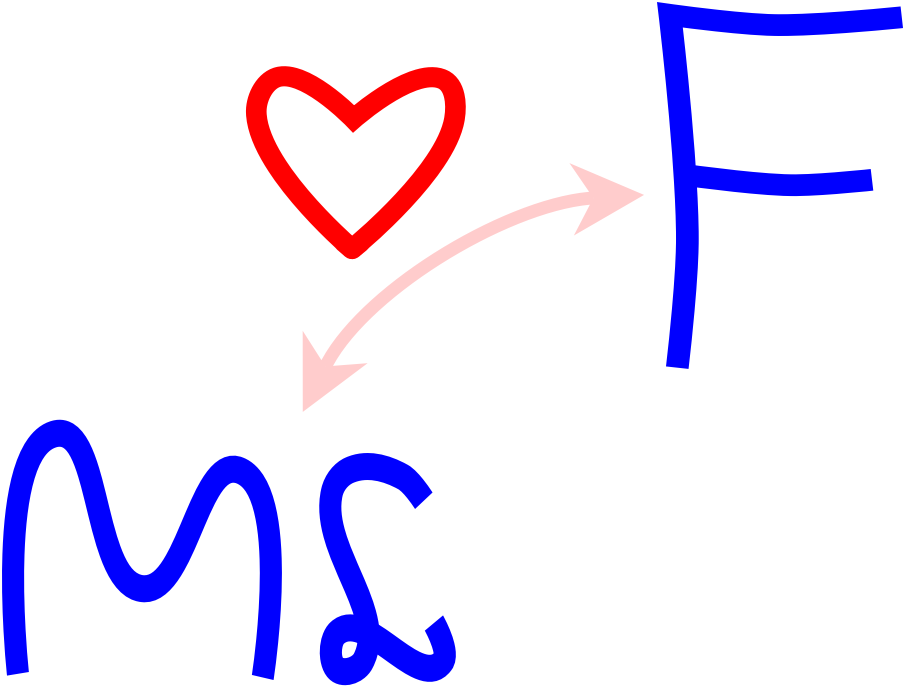
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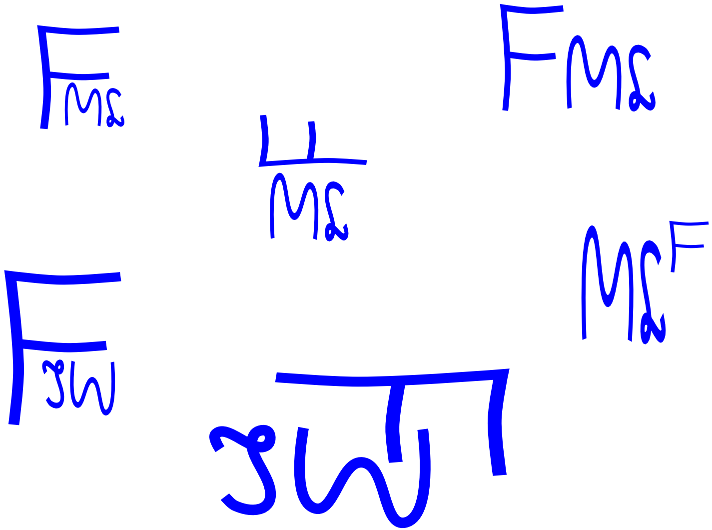

M&F

Outline

1. Design
   - *i*ML^F: an implicity-typed extension of System F
   - Types explained
   - *e*ML^F: an explicitly-typed version of *i*ML^F
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   - Examples
   - Type inference
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A universal type system

Explicit System F:

VAR
$$\frac{z : \tau \in \Gamma}{\Gamma \vdash z : \tau}$$

APP
$$\frac{\Gamma \vdash a_1 : \tau_2 \rightarrow \tau_1 \qquad \Gamma \vdash a_2 : \tau_2}{\Gamma \vdash a_1\ a_2 : \tau_1}$$

FUN
$$\frac{\Gamma, x : \tau_0 \vdash a : \tau}{\Gamma \vdash \lambda(x : \tau_0)\ a : \tau_0 \rightarrow \tau}$$

GEN
$$\frac{\Gamma, \alpha \vdash a : \tau_0}{\Gamma \vdash \Lambda\alpha.\ a : \forall(\alpha)\ \tau_0}$$

UNGEN
$$\frac{\Gamma \vdash a : \forall(\alpha)\ \tau}{\Gamma \vdash a\ \tau : \tau_0[\alpha \leftarrow \tau]}$$

A universal type system

Implicit System F:

$$\textsc{Var}\ \frac{z : \tau \in \Gamma}{\Gamma \vdash z : \tau} \qquad \textsc{App}\ \frac{\Gamma \vdash a_1 : \tau_2 \rightarrow \tau_1 \qquad \Gamma \vdash a_2 : \tau_2}{\Gamma \vdash a_1\ a_2 : \tau_1} \qquad \textsc{Fun}\ \frac{\Gamma, x : \tau_0 \vdash a : \tau}{\Gamma \vdash \lambda(x\quad)\ a : \tau_0 \rightarrow \tau}$$

$$\textsc{Gen}\ \frac{\Gamma, \alpha \vdash a : \tau_0}{\Gamma \vdash \qquad a : \forall(\alpha)\ \tau_0} \qquad \textsc{Ungen}\ \frac{\Gamma \vdash a : \forall(\alpha)\ \tau}{\Gamma \vdash a \quad : \tau_0[\alpha \leftarrow \tau]}$$

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$$\textsc{Gen}\quad \frac{\Gamma, \alpha \vdash a : \tau_0}{\Gamma \vdash \quad a : \forall(\alpha)\ \tau_0} \qquad \textsc{Inst}\quad \frac{}{\forall(\bar\alpha)\ \tau_0 \leqslant \tau_0[\bar\alpha \leftarrow \bar\tau]} \qquad \textsc{Sub}\quad \frac{\Gamma \vdash a : \tau_1 \qquad \tau_1 \leqslant \tau_2}{\Gamma \vdash a : \tau_2}$$

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$$\text{GEN}\quad \frac{\Gamma, \alpha \vdash a : \tau_0}{\Gamma \vdash \quad a : \forall(\alpha)\ \tau_0} \qquad \text{INST}\quad \frac{\bar\beta \notin \mathrm{ftv}(\forall(\bar\alpha)\ \bar\tau_0)}{\forall(\bar\alpha)\ \tau_0 \leqslant \forall(\bar\beta)\ \tau_0[\bar\alpha \leftarrow \bar\tau]} \qquad \text{SUB}\quad \frac{\Gamma \vdash a : \tau_1 \qquad \tau_1 \leqslant \tau_2}{\Gamma \vdash a : \tau_2}$$

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Add a construction for local bindings (perhaps derivable):

$$\textsc{Let}\ \frac{\Gamma \vdash a_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash a_2 : \tau}{\Gamma \vdash \text{let } x = a_1 \text{ in } a_2 : \tau}$$

A universal type system

Implicit System F:

VAR
$$\frac{z : \tau \in \Gamma}{\Gamma \vdash z : \tau}$$

APP
$$\Gamma \vdash$$

GEN
$$\frac{\Gamma, \alpha \vdash a : \tau_0}{\Gamma \vdash \quad a : \forall(\alpha)\ \tau_0}$$

Logical, canonical presentation of typing rules

- Covers many variations: F, ML, F^η, $F_\leq$, ...
  - Vary the set of types.
  - Vary the instance relation between types.
- For ML, just restrict types to ML types.

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$$\frac{\Gamma \vdash a_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash a_2 : \tau}{\Gamma \vdash \text{let } x = a_1 \text{ in } a_2 : \tau}$$

A universal type system

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$$\text{VAR}\quad \frac{z : \tau \in \Gamma}{\Gamma \vdash z : \tau}$$

$$\text{APP}\quad \Gamma \vdash$$

$$\text{GEN}\quad \frac{\Gamma, \alpha \vdash a : \tau_0}{\Gamma \vdash a : \forall(\alpha)\ \tau_0}$$

Logical, canonical presentation of typing rules

- Covers many variations: F, ML, F^η, $F_\leq$, ...
  - Vary the set of types.
  - Vary the instance relation between types.
- For ML, just restrict types to ML types.

Do never change the typing rules!

Add a construction for local bindings (perhaps derivable):

$$\text{LET}\quad \frac{\Gamma \vdash a_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash a_2 : \tau}{\Gamma \vdash \text{let } x = a_1 \text{ in } a_2 : \tau}$$

Type inference is undecidable — in System F

Of course, we must

- Use type annotations on function parameters in some cases.

When?

- Always?
  - too many annotations are obfuscating.
- Alleviate some annotations by local type inference?
  - unintuitive and fragile (to program transformations).
- When parameters have polymorphic types?
  - still two many bothersome type annotations.

Not conservative extensions of ML

Are polymorphic types less important than monorphic ones?

Type inference is undecidable — in System F

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Are polymorphic types less important than monorphic ones?

Our choice

explained below

- When (and only when) parameters are used polymorphically.

Lack of principal types for applications

The example of choice

let $\mathit{choice} = \lambda(x)\ \lambda(y)$ **if** true **then** x **else** $y : \forall\, \beta \cdot \beta \to \beta \to \beta$
let $\mathit{id} = \lambda(z)\ z : \forall(\alpha)\ \alpha \to \alpha$

$\mathit{choice}\ \mathit{id}$:

Lack of principal types for applications

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let $choice = \lambda(x)\ \lambda(y)$ **if** *true* **then** x **else** $y : \forall\, \beta \cdot \beta \to \beta \to \beta$
let $id = \lambda(z)\ z : \forall(\alpha)\ \alpha \to \alpha$

$$choice\ id : \begin{cases} \forall(\alpha)\ (\alpha \to \alpha) \to (\alpha \to \alpha) \\ (\forall(\alpha)\ \alpha \to \alpha) \to (\forall(\alpha)\ \alpha \to \alpha) \end{cases}$$

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$$\mathit{choice}\ \mathit{id} : \begin{cases} \forall(\alpha)\ (\alpha \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha) \\ (\forall(\alpha)\ \alpha \rightarrow \alpha) \rightarrow (\forall(\alpha)\ \alpha \rightarrow \alpha) \end{cases}$$

No better choice in F!

Lack of principal types for applications

The example of choice

let $\mathit{choice} = \lambda(x)\ \lambda(y)$ **if** true **then** x **else** $y : \forall\, \beta \cdot \beta \to \beta \to \beta$

let $\mathit{id} = \lambda(z)\ z : \forall(\alpha)\ \alpha \to \alpha$

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No better choice in F!

The problem is serious and inherent

- Follows from rules INST, GEN, and APP.
- Should values be kept as polymorphic or as instantiated as possible?
- A type inference system can do both, but cannot choose.

Lack of principal types for applications

The example of choice

let *choice* $= \lambda(x)\ \lambda(y)$ **if** *true* **then** x **else** $y : \forall\, \beta \cdot \beta \to \beta \to \beta$
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$$choice\ id : \begin{cases} \forall(\alpha)\ (\alpha \to \alpha) \to (\alpha \to \alpha) \\ (\forall(\alpha)\ \alpha \to \alpha) \to (\forall(\alpha)\ \alpha \to \alpha) \end{cases}$$

The solution in iML^F:

$$choice\ id : \quad \forall(\beta \geq \forall(\alpha)\ \alpha \to \alpha)\ \beta \to \beta$$

Lack of principal types for applications

The example of choice

let *choice* $= \lambda(x)\ \lambda(y)$ **if** *true* **then** x **else** $y : \forall\,\beta \cdot \beta \to \beta \to \beta$
let *id* $= \lambda(z)\ z : \forall(\alpha)\ \alpha \to \alpha$

$$\textit{choice id} : \left\{ \begin{array}{l} \forall(\alpha)\ (\alpha \to \alpha) \to (\alpha \to \alpha) \\ (\forall(\alpha)\ \alpha \to \alpha) \to (\forall(\alpha)\ \alpha \to \alpha) \end{array} \right.$$

The solution in iML^F:

$$\textit{choice id} : \quad {\color{red}\forall(\beta \geq} \forall(\alpha)\ \alpha \to \alpha{\color{red})\ \beta \to \beta}$$

$$\leqslant \left\{ \begin{array}{l} {\color{red}(\beta \to \beta)}\ [{\color{red}\beta} \leftarrow \forall(\alpha)\ \alpha \to \alpha] \\ \forall(\alpha)\ {\color{red}(\beta \to \beta)}\ [{\color{red}\beta} \leftarrow \alpha \to \alpha]) \end{array} \right.$$

The definition of *i*ML^F

Types are stratified

$$\begin{array}{rll} \sigma ::= & \tau & \in \mathsf{F} \\ \mid & \forall(\alpha \geq \sigma)\ \sigma & \end{array}$$

We can see and explain types by $\leqslant_{\mathsf{F}}$-closed sets of System-F types:

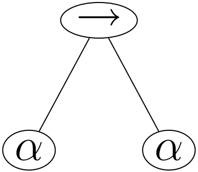
$$\begin{array}{rl} \{\!\{\tau\}\!\} & \triangleq \{\tau' \mid \tau \leqslant_{\mathsf{F}} \tau'\} \\ \{\!\{\forall(\alpha \geq \sigma)\ \sigma'\}\!\} & \triangleq \left\{ \forall(\bar{\beta})\ \tau'[\alpha \leftarrow \tau] \mid \wedge \left(\begin{array}{l} \tau \in \{\!\{\sigma\}\!\} \wedge \tau' \in \{\!\{\sigma'\}\!\} \\ \bar{\beta} \# \mathsf{ftv}(\forall(\alpha \geq \sigma)\ \sigma') \end{array} \right. \right\} \end{array}$$

Type instance $\leqslant_I$ is set inclusion on the translations

$$\sigma \leqslant_I \sigma' \iff \{\!\{\sigma\}\!\} \supseteq \{\!\{\sigma'\}\!\}$$

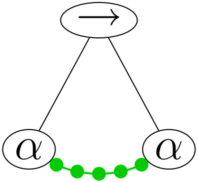
Simple types

$$\alpha \to \alpha$$



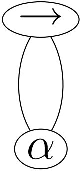
Simple types

$$\alpha \to \alpha$$



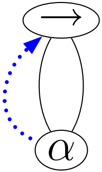
Simple types

$$\alpha \to \alpha$$



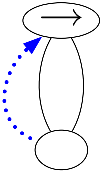
System-F types

$$\forall(\alpha)\ \alpha \to \alpha$$



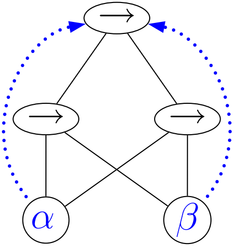
System-F types

$$\forall(\alpha)\ \alpha \to \alpha$$



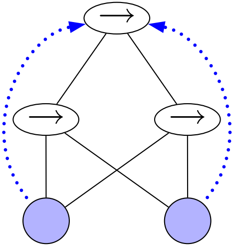
System-F types

$$\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to \alpha \to \beta$$



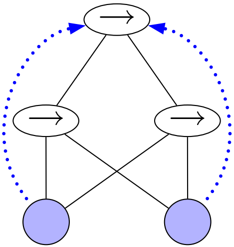
System-F types

$$\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to \alpha \to \beta$$

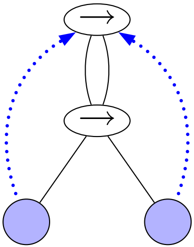


System-F types

$$\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to \alpha \to \beta$$

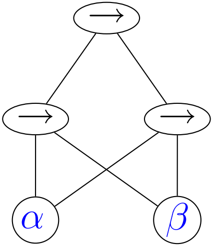


$\lessdot_F$

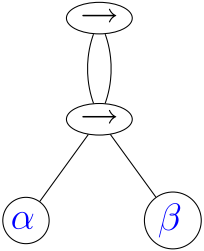


System-F types

$$(\alpha \to \beta) \to \alpha \to \beta$$



$\sqsupseteq ST$

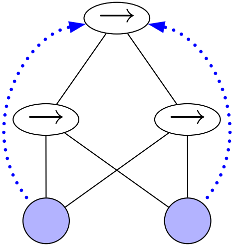


Sharing of inner nodes:

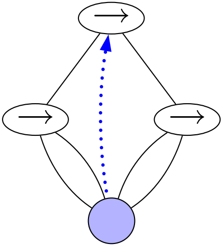
- Coming from the dag-representation of simple types.
- Canonical (unique) representation if disallowed.

System-F types

$$\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to \alpha \to \beta$$



$\leqslant_F$

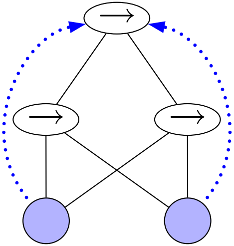


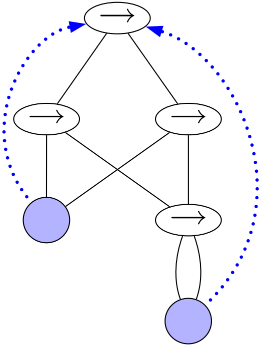
$$\forall(\alpha)\ (\alpha \to \alpha) \to \alpha \to \alpha$$

System-F types

$$\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to \alpha \to \beta$$

$\leqslant_F$



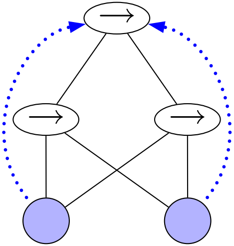


$$\forall(\alpha)\ \forall(\gamma)\ (\alpha \to \gamma \to \gamma) \to \alpha \to \gamma \to \gamma$$

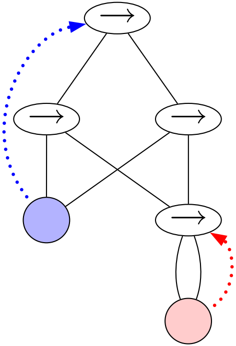
System-F types

more

$$\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to \alpha \to \beta$$



$\leqslant_F$



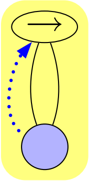
$$\forall(\alpha)(\alpha \to \forall(\gamma)\ \gamma \to \gamma) \to \alpha \to \forall(\gamma)\ \gamma \to \gamma$$

Types in $i\mathsf{ML}^{\mathsf{F}}$

$$\forall(\beta \geq \forall(\alpha)\, \alpha \to \alpha\,) \to \beta \to \beta$$

Types in iML^F

$$\forall(\beta \geq \forall(\alpha)\ \alpha \to \alpha) \to \beta \to \beta$$



Types in iML^F

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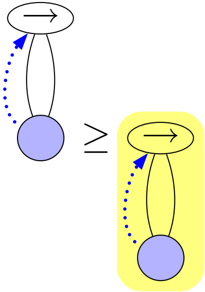

Types in iML^F

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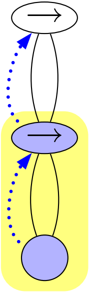

Types in iML^F

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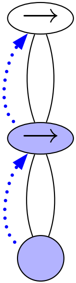
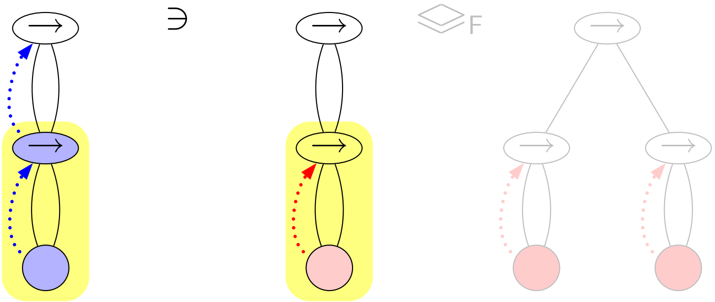


Types in iML^F

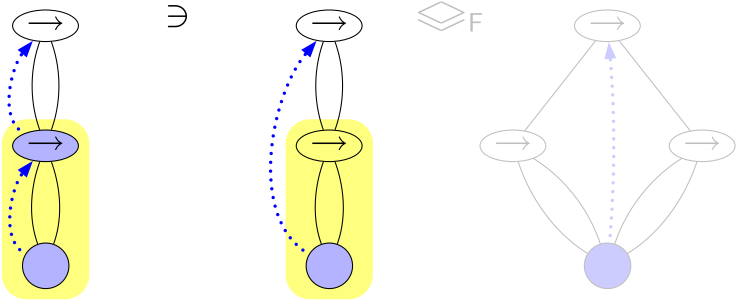
$$\forall(\beta \geq \forall(\alpha)\, \alpha \to \alpha)\to \beta \to \beta$$



$$(\forall(\alpha)\, \alpha \to \alpha) \to \forall(\alpha)\, \alpha \to \alpha$$

Types in iML^F

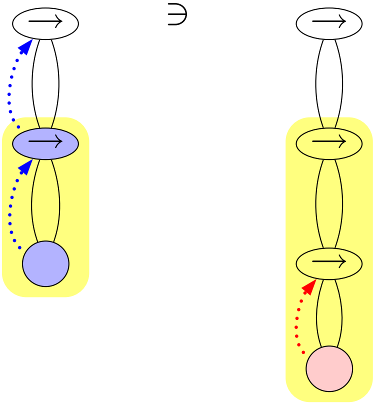
$$\forall(\beta \geq \forall(\alpha)\ \alpha \to \alpha\ ) \to \beta \to \beta$$



$$\forall(\alpha)\ (\alpha \to \alpha) \to (\alpha \to \alpha)$$

Types in iML^F

$$\forall(\beta \geq \forall(\alpha)\, \alpha \to \alpha) \to \beta \to \beta$$



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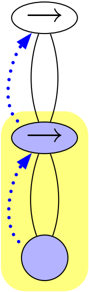

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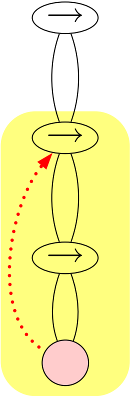

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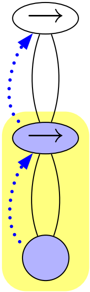
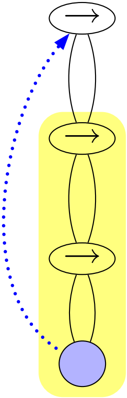
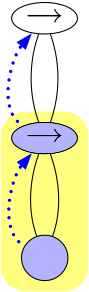


$\ni$



Types in iML^F

$$\forall(\beta \geq \forall(\alpha)\ \alpha \to \alpha\ ) \to \beta \to \beta$$



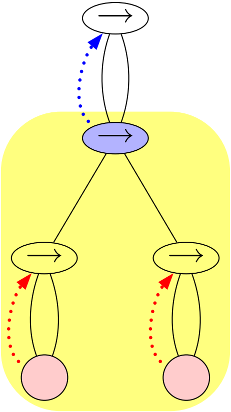
$\ni$ . . .

The semantics cannot be captured by

- a finite set of System-F types up to $\leqslant$
- a finite intersection type.

iML^F types

$$\forall(\beta \geq (\forall(\alpha)\, \alpha \to \alpha) \to (\forall(\alpha)\, \alpha \to \alpha)) \to \beta \to \beta$$



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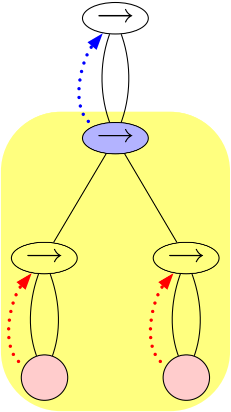
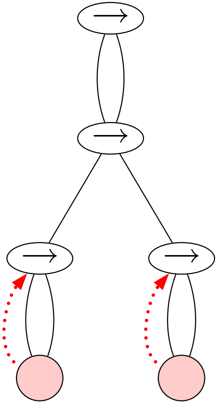


$\ni$

only



iML^F types

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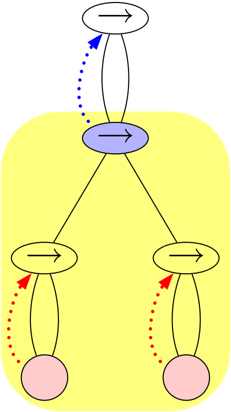
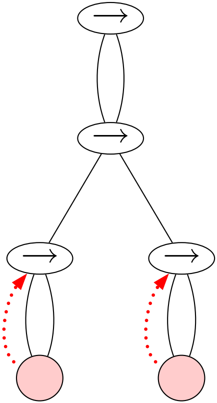


$\sqsubseteq_I$



iML^F types

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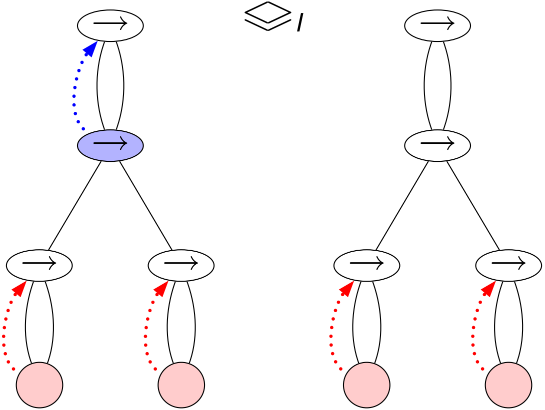
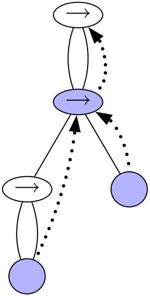


Type instance $\leqslant$ in iML^F

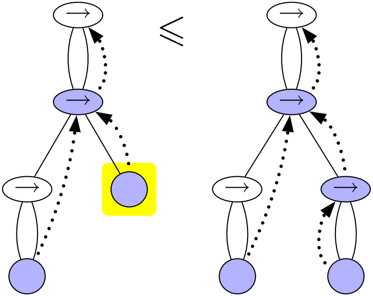
Only four atomic instance operations, and only two new.



Type instance $\leqslant$ in iML^F

Only four atomic instance operations, and only two new.

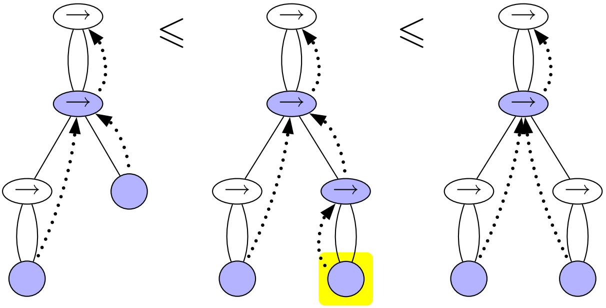
Grafting



Type instance $\leqslant$ in iML^F

Only four atomic instance operations, and only two new.

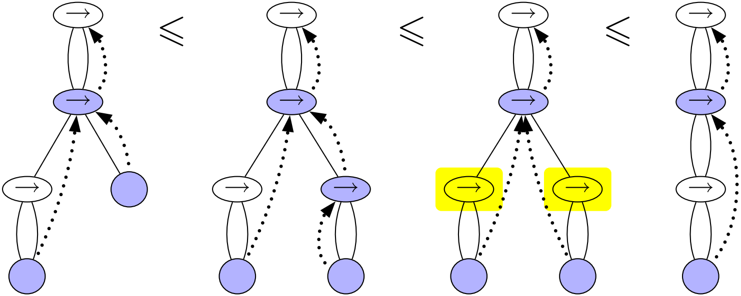
Grafting Raising



Type instance $\leqslant$ in $i\mathsf{ML}^\mathsf{F}$

Only four atomic instance operations, and only two new.

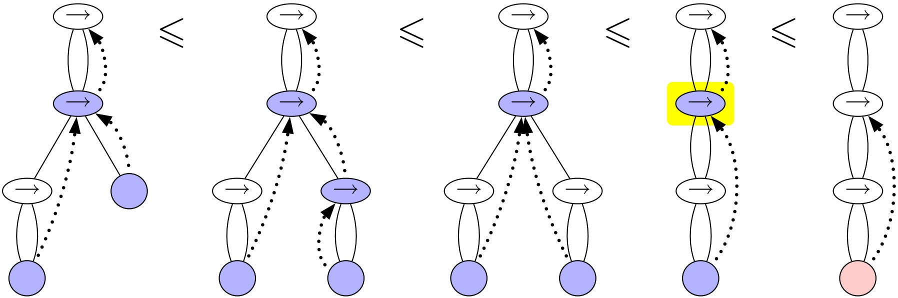
Grafting Raising Merging



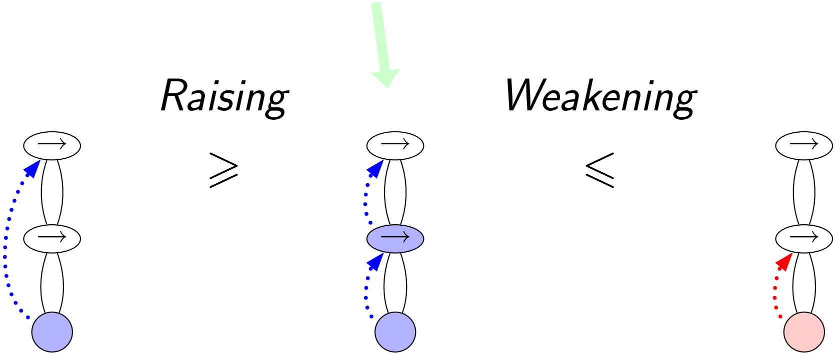
Type instance $\leqslant$ in iML^F

Only four atomic instance operations, and only two new.

Grafting Raising Merging Weakening



Checking the example choice id



Outline

1. Design
   - iML^F: an implicity-typed extension of System F
   - Types explained
   - eML^F: an explicitly-typed version of iML^F

2. Uses and Implementation
   - Examples
   - Type inference
   - Restrictions and extensions

Design of $e\mathrm{ML}^{\mathrm{F}}$

Goal

Find a restriction $i\mathrm{ML}^{\mathrm{F}}$ where programs that would require guessing polymorphism are ill-typed.

Guideline

◂ design

Function parameters that are used polymorphically (and only those) need an annotation.

First-order inference with second-order types

Easy examples

$\lambda(z)\ z$	:	$\forall(\alpha)\ \alpha \to \alpha$	as in ML
let $x = \lambda(z)\ z$ in $x\ x$	:	$\forall(\alpha)\ \alpha \to \alpha$	as in ML
$\lambda(x)\ x\ x$	:	ill-typed!	x is used polymorphically
$\lambda(x : \forall(\alpha)\ \alpha \to \alpha)\ x\ x$	:	$(\forall(\alpha)\ \alpha \to \alpha) \to (\forall(\alpha)\ \alpha \to \alpha)$	

First-order inference of second order types

More challenging example

$(\lambda(z)\ z)\ \ (a : \sigma)$ where σ is truly polymorphic

First-order inference of second order types

More challenging example

$$(\lambda(z)\ z)\ \ (a : \sigma) \quad \text{where } \sigma \text{ is truly polymorphic}$$

- z must carry values of a polymorphic type.
- but z is not used polymorphically.
- Indeed, it can be typed in System F as n

$$(\Lambda\alpha.\ \lambda(z : \alpha)\ z)\ [\sigma]\ (a : \sigma)$$

First-order inference of second order types

More challenging example

$(\lambda(z)\ z)\ \ (a : \sigma)$ where σ is truly polymorphic **ACCEPT**

- z must carry values of a polymorphic type.
- but z is not used polymorphically.
- Indeed, it can be typed in System F as n

$$(\Lambda\alpha.\ \lambda(z : \alpha)\ z)\ [\sigma]\ (a : \sigma)$$

First-order inference of second order types

More challenging example

$$\lambda(z)\ (z\ (a : \sigma))$$

First-order inference of second order types

More challenging example

$$\lambda(z)\ (z\ (a : \sigma))$$

- z have the polymorphic type $\sigma \to \sigma$?
- z is node used polymorphically:
  polymorphism is only carried out from the argument to the result.

First-order inference of second order types

More challenging example

$$\lambda(z)\ (z\ (a : \sigma))$$

ACCEPT

- z have the polymorphic type $\sigma \to \sigma$?
- z is node used polymorphically:
  polymorphism is only carried out from the argument to the result.

Abstracting second-order polymorphism as first-order types

Solution

1) Disallow second-order types under arrows, *e.g.* such as $\sigma_{id} \to \sigma_{id}$
2) Instead, allow type variables to stand for polymorphic types:

write	$\forall(\alpha \Rightarrow \sigma_{id})\ \alpha \to \alpha$
read	"$\alpha \to \alpha$ where α abstracts σ_{id}"
means	$\sigma_{id} \to \sigma_{id}$

Mechanism

1) function parameters must be monomorphic (but may be abstract).
2) forces all polymorphism to be abstracted away in the context.

$$\cfrac{\cfrac{\alpha \Rightarrow \sigma_{id}, x : \alpha \vdash x : \alpha}{\alpha \Rightarrow \sigma_{id} \vdash \lambda(x)\ x : \alpha \to \alpha}}{\lambda(x)\ x : \forall(\alpha \Rightarrow \sigma_{id})\ \alpha \to \alpha}$$

Abstracting second-order polymorphism

more

Key point: abstraction is directional

$$\alpha \Rightarrow \sigma \vdash \sigma \leqslant \alpha \qquad\qquad \alpha \Rightarrow \cancel{\sigma \vdash} \alpha \leqslant \sigma$$

Hence,

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdash a : \sigma \qquad \cdot}{\alpha \Rightarrow \sigma \vdash a : \alpha} \qquad \alpha \Rightarrow \sigma,\ z : \alpha \to \alpha \vdash z : \alpha \to \alpha}{\alpha \Rightarrow \sigma,\ z : \alpha \to \alpha \vdash z\ a : \alpha}}{\alpha \Rightarrow \sigma \vdash \lambda(z)\ z\ a : (\alpha \to \alpha) \to \alpha}}{\vdash \lambda(z)\ z\ a : \forall(\alpha \Rightarrow \sigma)\ (\alpha \to \alpha) \to \alpha}$$

Abstracting second-order polymorphism

more

Key point: abstraction is directional

$$\alpha \Rightarrow \sigma \vdash \sigma \leqslant \alpha \qquad\qquad \cancel{\alpha \Rightarrow \sigma \vdash \alpha \leqslant \sigma}$$

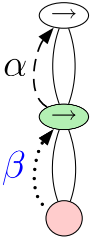
But,

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\alpha \Rightarrow \sigma_{\mathsf{id}},\ z : \alpha \vdash z : \alpha}{\alpha \Rightarrow \sigma_{\mathsf{id}},\ z : \alpha \vdash z : \sigma_{\mathsf{id}}}}{\alpha \Rightarrow \sigma_{\mathsf{id}},\ z : \alpha \vdash z : \alpha \to \alpha} \qquad \alpha \Rightarrow \sigma_{\mathsf{id}},\ z : \alpha \vdash z : \alpha}{\alpha \Rightarrow \sigma_{\mathsf{id}},\ z : \alpha \vdash z\ z : \alpha}}{\alpha \Rightarrow \sigma_{\mathsf{id}} \vdash \lambda(z)\ z\ z : \alpha \to \alpha}}{\vdash \lambda(z)\ z\ z : \forall(\alpha \geq \sigma_{\mathsf{id}})\ \alpha \to \alpha}$$

Types in eML^F

Introduce a new binder for abstraction

$$\forall(\alpha \Rightarrow \forall(\beta)\ \beta \to \beta)\ \alpha \to \alpha$$



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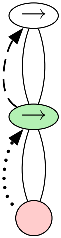
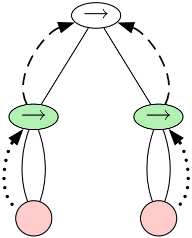


Types in eML^F

Introduce a new binder for abstraction

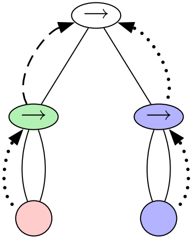
$$\forall(\alpha \Rightarrow \forall(\beta)\ \beta \to \beta)\ \forall(\alpha' \Rightarrow \forall(\beta)\ \beta \to \beta)\ \alpha \to \alpha'$$



Types in eML^F

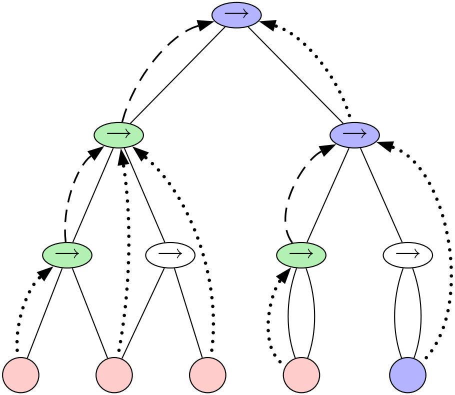
Introduce a new binder for abstraction

$$\forall(\alpha \Rightarrow \forall(\beta)\ \beta \to \beta)\ \forall(\alpha' \geq \forall(\beta)\ \beta \to \beta)\ \alpha \to \alpha'$$



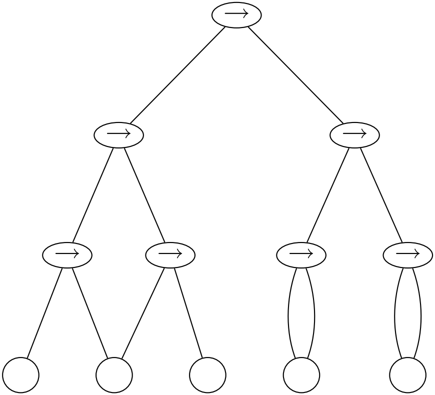
Types, graphically

= first-order term-dag + a binding tree



Types, graphically

$=$ first-order term-dag $+$ a binding tree



Types, graphically

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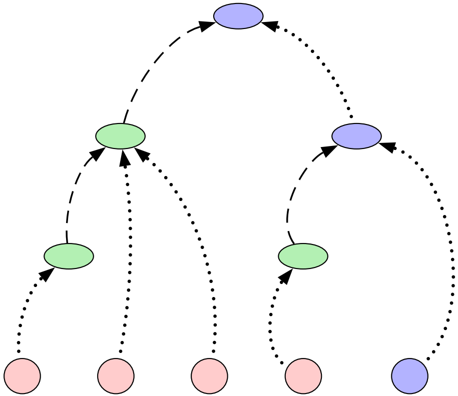
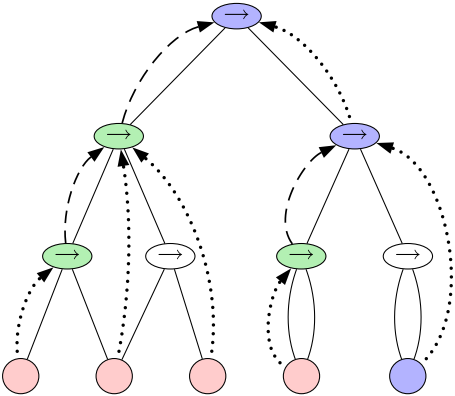


Types, graphically

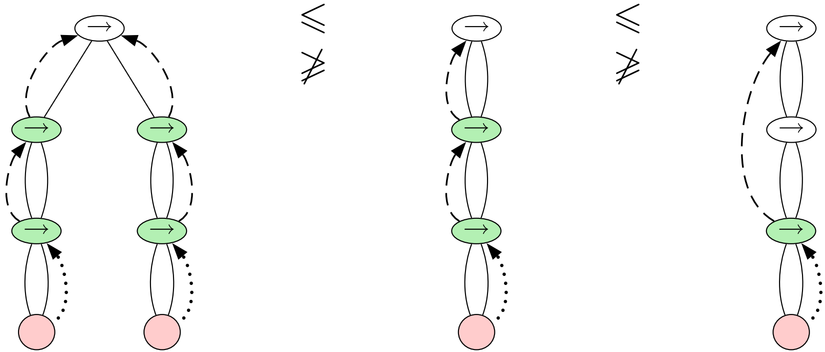
= first-order term-dag + a binding tree



+ well-formedness conditions relating the two

Type instance $\leqslant$ in eML^F

Sharing and binding of abstract nodes matter



Grafting, Merging, Raising, Weakening

Unchanged.

Type annotations

Recovering the missing power

$$(\leqslant) \subset (\leqslant_I)$$

- $\leqslant$ is weaker than $\leqslant_I$, as sharing and binding of abstract nodes matters.

Type annotations

Recovering the missing power

$$(\leqslant) \subset (\leqslant_I) = (\leqslant \cup \mathrel{\diamond}_I)^* = (\leqslant ; \mathrel{\diamond}_I)$$

- $\leqslant$ is weaker than $\leqslant_I$, as sharing and binding of abstract nodes matters.
- Use explicit type annotations to recover $(\mathrel{\diamond}_I \setminus \leqslant)$.

  Notice that the weaker $\leqslant$, the more annotations will be required.

Type annotations

Recovering the missing power

$$(\leqslant) \subset (\leqslant_I) = (\leqslant \cup \otimes_I)^* = (\leqslant ; \otimes_I)$$

- Intuitively,

$$\frac{\Gamma \vdash a : \tau \qquad \tau \otimes_I \tau'}{\Gamma \vdash (a : \tau') : \tau'}$$

- Actually, use coercion functions:

$$(\_ : \quad \sigma) \;:\; \forall(\alpha \Rightarrow \sigma)\, \forall(\alpha' \Rightarrow \sigma)\, \alpha \to \alpha'$$

- Add syntactic sugar $\lambda(x : \sigma)\, a \;\stackrel{\triangle}{=}\; \lambda(x)\, \text{let } x = (x : \sigma) \text{ in } a \;\equiv\; \lambda(x)\, a[x \leftarrow (x : \sigma)]$

Type annotations

Recovering the missing power

$$(\leqslant) \subset (\leqslant_I) = (\leqslant \cup \mathrel{\diamondsuit}_I)^* = (\leqslant ; \mathrel{\diamondsuit}_I)$$

- Intuitively,

$$\frac{\Gamma \vdash a : \tau \qquad \tau \mathrel{\diamondsuit}_I \tau'}{\Gamma \vdash (a : \tau') : \tau'}$$

- Actually, use coercion functions:

$$(\_ : \exists\,(\bar\beta)\;\sigma) \;:\; \forall(\bar\beta)\,\forall(\alpha \Rightarrow \sigma)\,\forall(\alpha' \Rightarrow \sigma)\;\alpha \to \alpha'$$

- Add syntactic sugar $\lambda(x : \sigma)\; a \;\stackrel{\triangle}{=}\; \lambda(x)\;\mathrm{let}\; x = (x : \sigma)\;\mathrm{in}\; a$
  $\equiv\; \lambda(x)\; a[x \leftarrow (x : \sigma)]$

Type annotations

Remember $\alpha \Rightarrow \sigma, x : \alpha \vdash x : \sigma$

- Prevents typing $\lambda(x)\ x\ x$

With an annotation $\alpha \Rightarrow \sigma, x : \alpha \vdash (x : \sigma) : \sigma$ ▸ more

- Allows typing $\lambda(x : \sigma_{\mathsf{id}})\ x\ x$

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About principal types

Fact

- Programs have principal types, given with their type annotations.

Programs with type annotations

- Two versions of the same program, but with different type annotations, usually have different principal types.

Programs typable without type annotations

- Exactly ML programs.
- But usually have a more general type than in ML (*e.g. choice id*)
- Annotations may still be useful to get more polymorphism.

Robustness to small program transformations

Agreed

- Programmmers must be free of choising their programming patterns/styles.
- Programs should be maintainable.

Therefore

- Programs should be stable under some small, but important program transformations.

Robustness to small program transformations

$a \subseteq a'$ means *all typings of a are typings of a'*

Let-conversion

$\text{let } x = a_1 \text{ in } a_2 \;\subseteq\!\!\supseteq\; a_2[x \leftarrow a_1]$

Common subexpression can be factored out.

Redefine application

$a_1\ a_2 \;\subseteq\!\!\supseteq\; (\lambda(f)\ \lambda(x)\ f\ x)\ a_1\ a_2$

Many functionals, such as maps are typed as applications.

η-conversion of functional expressions

$a \;\subseteq\!\!\supseteq\; \lambda(x)\ a\ x$

Delay the evaluation.

Reordering of arguments

$a\ a_1\ a_2 \;\subseteq\!\!\supseteq\; (\lambda(x)\ \lambda(y)\ a\ y\ x)\ a_2\ a_1$

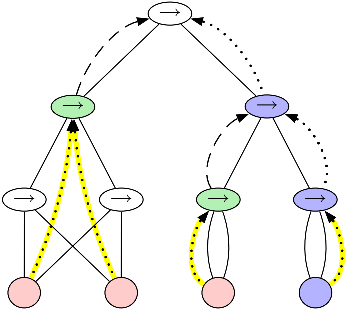
Curryfication

$a\ (a_1, a_2) \;\subseteq\!\!\supseteq\; (\lambda(x)\ \lambda(y)\ a\ (x, y))\ a_1\ a_2$

All valid in ML^F

Printing types

Only overlined bindings need to be drawn



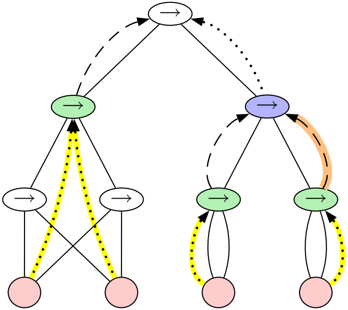
Leave implicit bindings that are

- at unshared, inner nodes,
- bound just above,
- abstractions on the left of arrows,
- instances on the right arrows.

$$(\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to (\alpha \to \beta)) \to (\forall(\alpha)\ \alpha \to \alpha) \to (\forall(\alpha)\ \alpha \to \alpha)$$

Printing types

Only overlined bindings need to be drawn



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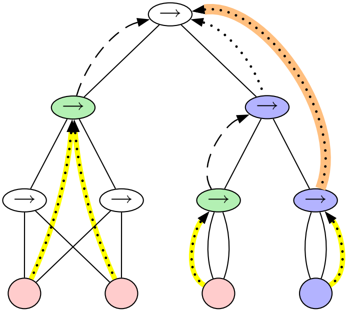
$$(\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to (\alpha \to \beta)) \to \forall(\gamma \Rightarrow \forall(\alpha)\ \alpha \to \alpha)\ (\forall(\alpha)\ \alpha \to \alpha) \to \gamma$$

Printing types

Only overlined bindings need to be drawn



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$$\forall(\gamma \Rightarrow \forall(\alpha)\ \alpha \to \alpha)\ (\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to (\alpha \to \beta)) \to (\forall(\alpha)\ \alpha \to \alpha) \to \gamma$$

Printing types

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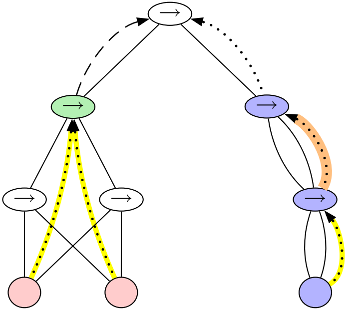

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- instances on the right arrows.

$$(\forall(\alpha)\ \forall(\beta)\ (\alpha \to \beta) \to (\alpha \to \beta)) \to \forall(\gamma \geq \sigma_{\mathsf{id}})\ \gamma \to \gamma$$

Examples

Library functions

```
let rec fold f v = function
  | Nil → v
  | Cons (h, t) → fold f (f h t) t ;;
```

val fold : $\forall(\alpha)\,\forall(\beta)\,(\alpha \to \alpha\ \mathit{list} \to \beta) \to \beta \to \alpha\ \mathit{list} \to \beta$

Few type annotations are needed in practice

- No dummmy/annoying/unpredictable annotations.

Output types are usually readable

- Most inner binding edges may be left implicit.
- Many library functions libraries keep their ML type in ML^{F}, modulo the syntactic sugar.

More examples

skip

Church's numerals

type nat $= \forall\ (\alpha)\ (\alpha \to \alpha) \to \alpha \to \alpha$;;
let zero $=$ fun f x $\to$ x;;
val zero : $\forall(\alpha)\ \alpha \to (\forall(\beta)\ \beta \to \beta)$

With type annotations on the iterator

let succ (n : nat) = fun f x $\to$ n f (f x);;
val succ : nat $\to (\forall\ (\alpha)\ (\alpha \to \alpha) \to \alpha \to \alpha)$

let add (n : nat) m = n succ m;;
val add : nat $\to (\forall\ (\alpha)\ (\alpha \to \alpha) \to \alpha \to \alpha)$

let mul n (m : nat) = m (add n) zero;;
mul : nat $\to$ nat $\to (\forall(\alpha)\ (\alpha \to \alpha) \to \alpha \to \alpha)$

More examples

skip

Church's numerals

type nat $= \forall\ (\alpha)\ (\alpha \to \alpha) \to \alpha \to \alpha$;;
let zero $=$ fun f x $\to$ x;;
val zero : $\forall(\alpha)\ \alpha \to (\forall(\beta)\ \beta \to \beta)$

Without type annotations

let succ n $=$ fun f x $\to$ n f (f x);;
val succ : $\forall\ (\alpha, \beta, \gamma)\ ((\alpha \to \beta) \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$
let add n m $=$ n succ m;;
val add : $\forall(\delta \geq \forall(\alpha,\beta,\gamma)\ ((\alpha \to \beta) \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma)$
$\forall(\varepsilon,\varphi)\ (\delta \to \varepsilon \to \varphi) \to \varepsilon \to \varphi$

In ML:

val add : $\forall\ (\alpha,\beta,\gamma,\varepsilon,\varphi)\ ((((\alpha \to \beta) \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma)$
$\to \varepsilon \to \varphi) \to \varepsilon \to \varphi$

More examples

skip

Church's numerals

type nat $= \forall\,(\alpha)\;(\alpha \to \alpha) \to \alpha \to \alpha$;;
let zero $=$ fun f x $\to$ x;;
val zero : $\forall(\alpha)\,\alpha \to (\forall(\beta)\,\beta \to \beta)$

Mandatory type annotations

let mul n m $=$ m (add n) zero;;
let mul' $=$ (mul : : nat $\to$ nat $\to$ nat);;
fails

ML^{F} without any type annotation at all does not do better than ML!

Unification algorithm

Computes principal unifiers, in three steps

- Computes the underlying dag-structure by first-order unification.
- Computes the binding structure
  - by raising binding edges
  - as little as possible to maintain well-formedness.
- Checks that no locked binding edge (in red) has been raised or merged.

Complexity

- Same as first-order unification. Other passes are in linear time.
- $O(n)$ (or $O(n\alpha(n))$ if incremental).

Note

- The algorithm performs "*first-order unification of second-order types*".

Type inference

Proceeds much as in ML

- Implement type-instantiation by copying the polymorphic part.
- Use unification to solve typing constraints.
- Generalize as much as possible at every step (not just at every let).

Type inference with typing constraints

Demo

Complexity in $O(kn(\alpha(kn)+d)) \approx O(kdn)$

- As for ML (see McAllester).
  - k is the maximal size of types (usually not too large)
  - d is the maximal nesting of type schemes.
- However, ML and ML^{F} differs on d, which is:
  - the left-nesting of let-bindings in ML
  - the maximun height of an expression in ML^{F} (Still, does not grow on the right of let-bindings).

Variations on ML^F

Shallow ML^F

The version we presented is a "downgraded" version of ML^F.

- Types are stratified.
- Instance bounded types cannot appear in bounds of abstract variables.
- In particular, type annotations must be F types.

Variations on ML^F

Shallow ML^F

The version we presented is a "downgraded" version of ML^F.

- Types are stratified.
- Instance bounded types cannot appear in bounds of abstract variables.
- In particular, type annotations must be F types.

Full ML^F

- No stratification, more expressive.
- All interesting properties are preserved.
- Algorithms are mostly unchanged.
- We loose the interpretation of types as sets of System-F types.

Variations on ML^F

Shallow ML^F

The version we presented is a "downgraded" version of ML^F.

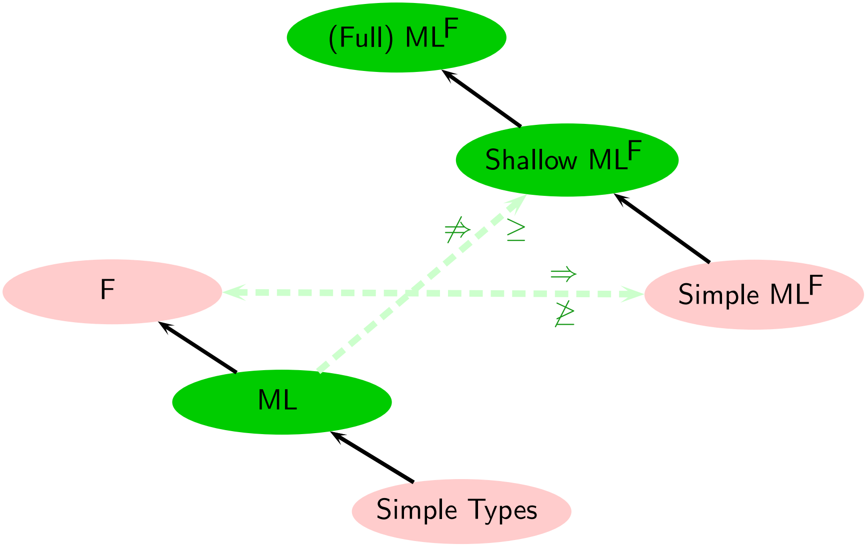
- Types are stratified.
- Instance bounded types cannot appear in bounds of abstract variables.
- In particular, type annotations must be F types.

Simple ML^F

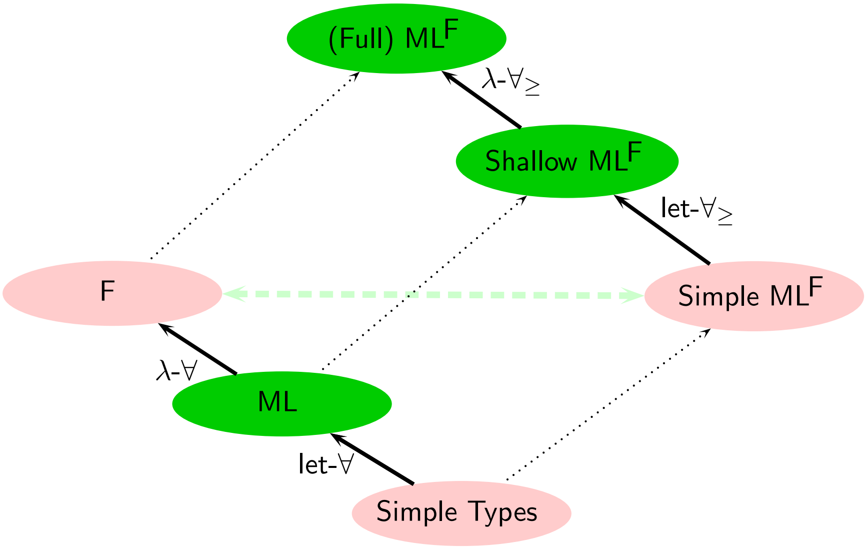
Remove instance bindings $\geq$, keep abstract bindings $\Rightarrow$.

- Equivalent to System F.
- Principal types are lost (no type inference).

A hierarchy of languages

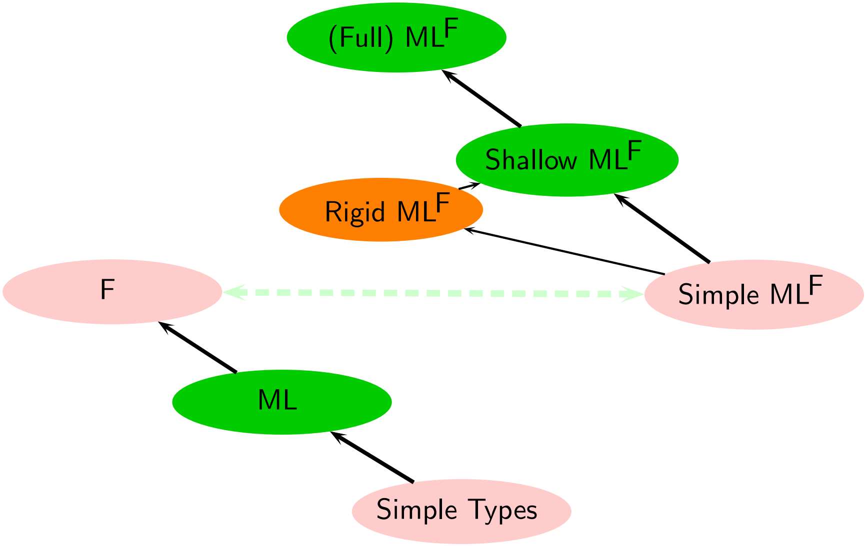


A hierarchy of languages



A hierarchy of languages

▸ Rigid ML^F



Extensions

Primitive Existential types

- Encoding with existential types works well (only annotate at creation).
- Can more be done with primitive existential ?

(Equi-) recursive types

- Easy when cycles do not contain quantifiers.
- Cycles that croses quantifiers are difficult.

Higher-order types

- Use two quantifiers (explicit coercions between the two permitted)
  - $\forall^{\mathsf{F}}$ for fully explicit type abstractions and
  - $\forall^{\mathsf{ML^F}}$ for implicit $\mathsf{ML^F}$ polymorphism.
- Restrict $\forall^{\mathsf{ML^F}}$ to the first-order type variables.
- Can $\forall^{\mathsf{ML^F}}$ also be used at higher-order kinds?

Papers and prototypes

Talk mainly based on

- Recasting-ML^F *with Didier Le Boltan*.
- A Graphical Presentation of ML^F Types, *with Boris Yakobowski*.

Other papers and online prototype at

- http://gallium.inria.fr/˜remy/mlf/

See also Daan Leijen's papers and prototypes

- http://research.microsoft.com/users/daan/pubs.html

Conclusions

Just two things to remember

- ML^{F} allows function parameters to implicitly carry polymorphic values that are used monomorphically.
- Type annotations are required only to allow function parameters to carry (polymorphic) values that are used polymophically.

ML^{F} design, use, and implementation are close to ML

- ML^{F} piggy-backs on ML type-shemes and generalization mechanism.
- Part of the credits should be returned to the great designers of ML.

Hopefully

- ML users will feel "*at home*".
- Other users will also appreciate the convenience of type inference.

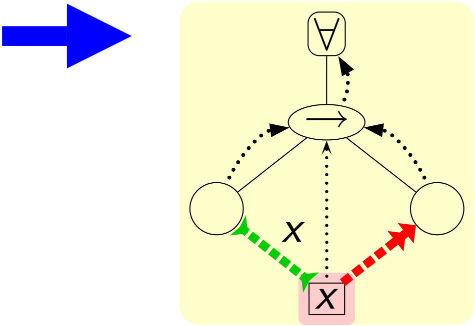
Appendix

Type inference with typing constraints (demo)

$$\boxed{\lambda(x)\ x}$$

Type inference with typing constraints (demo)

$\boxed{\lambda(x)\ x}$



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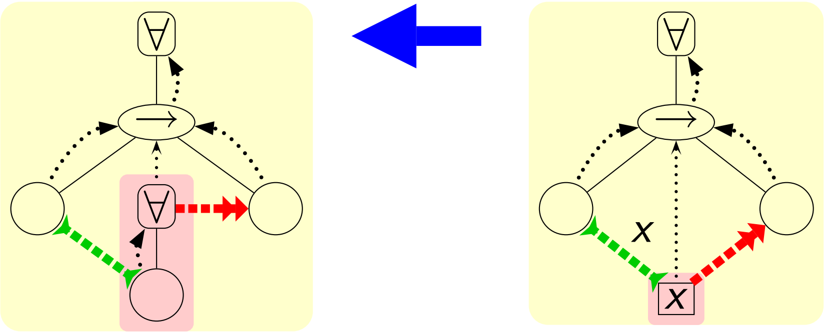

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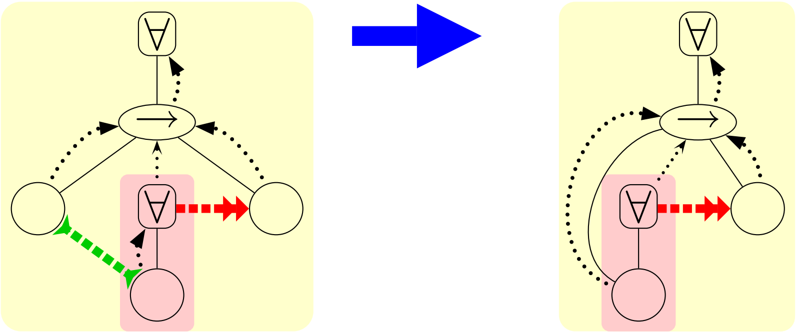

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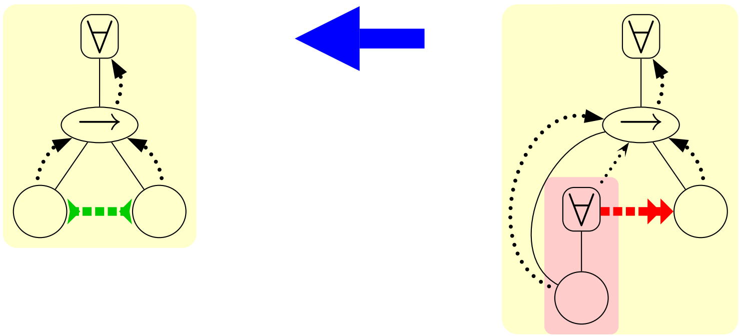

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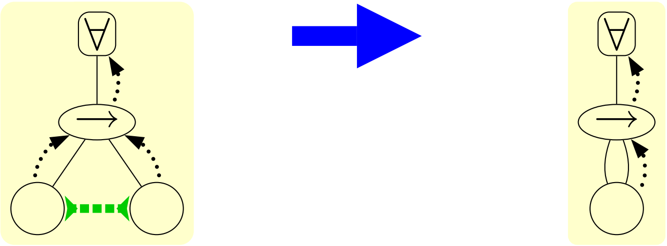

Type inference with typing constraints (demo)

skip back

$$\boxed{\begin{array}{c}\text{let } y = \lambda(x)\ x \\ \text{in } y\ y\end{array}}$$

Type inference with typing constraints (demo)

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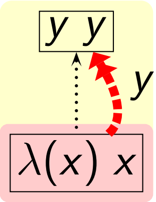

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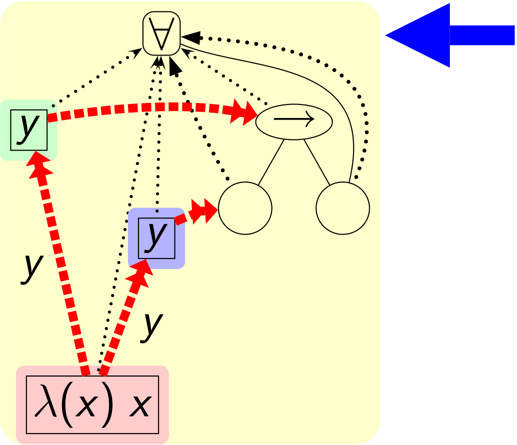

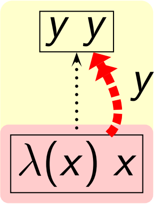

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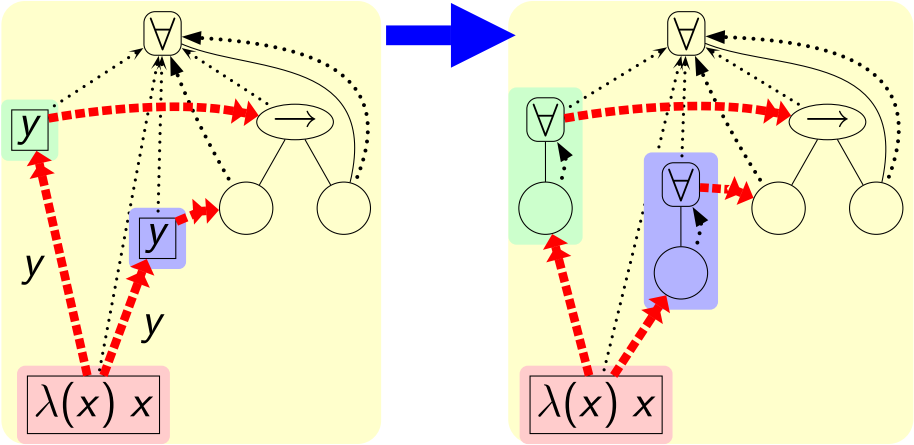

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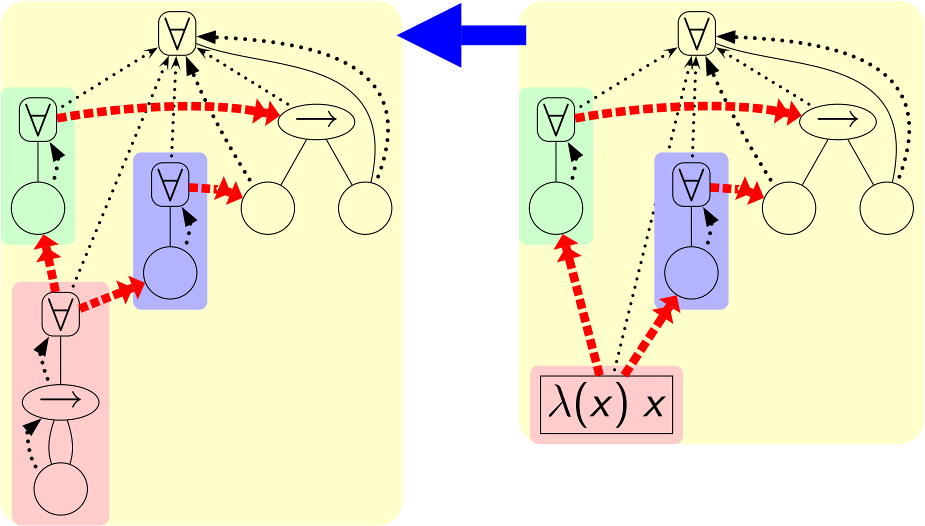

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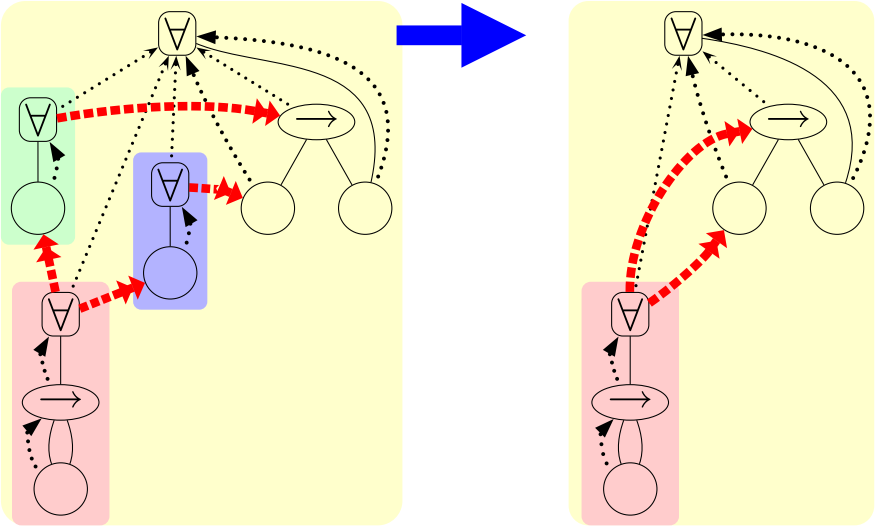

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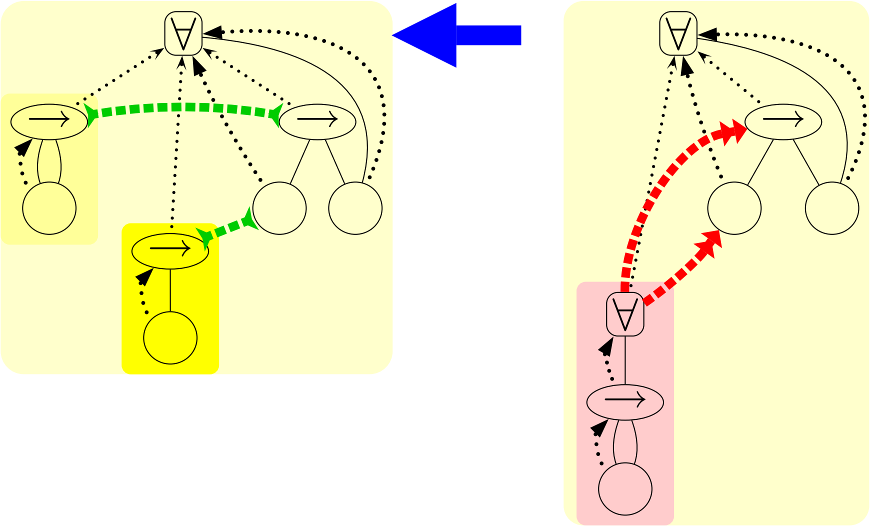

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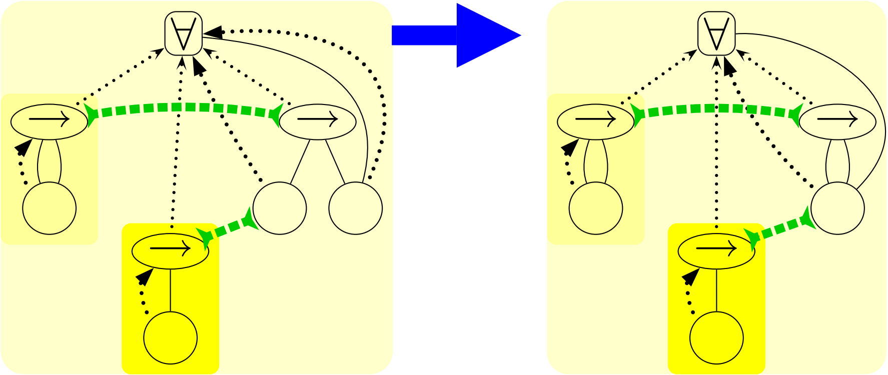

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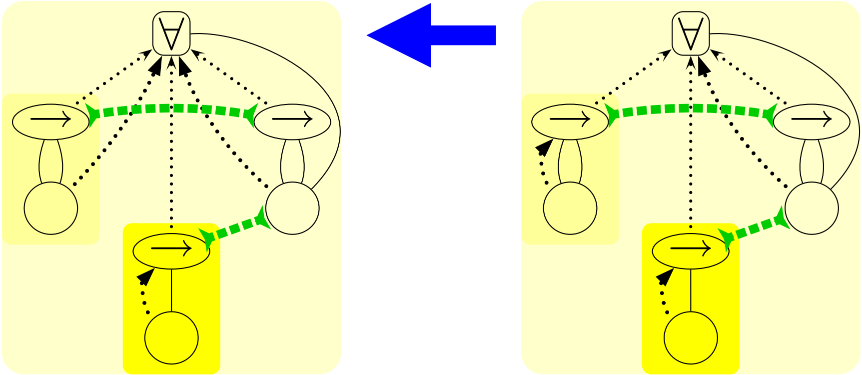

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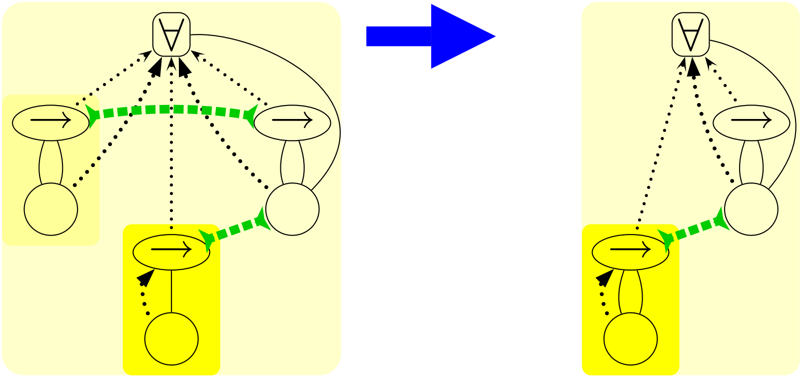

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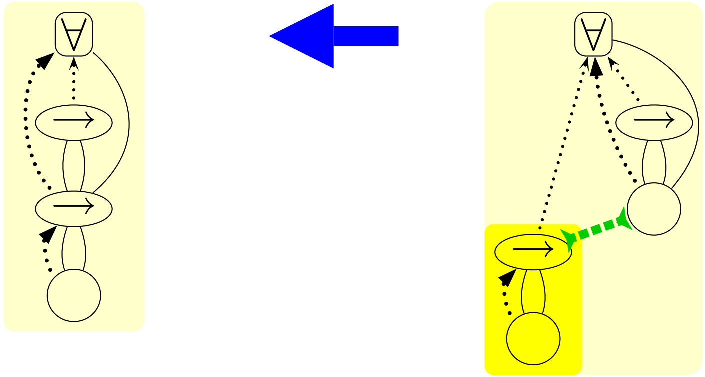

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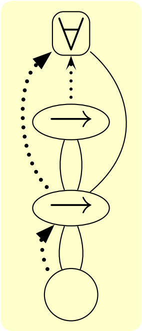
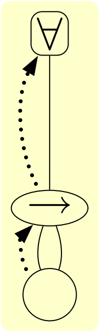




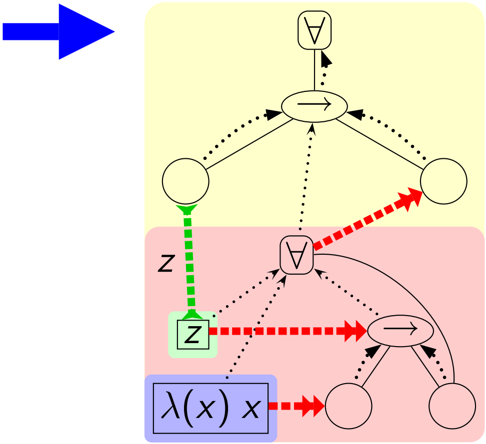
Type inference with typing constraints (demo)

skip back

$$\boxed{\lambda(z)\ z\ (\lambda(x)\ x)}$$

Type inference with typing constraints (demo)

$\boxed{\lambda(z)\ z\ (\lambda(x)\ x)}$



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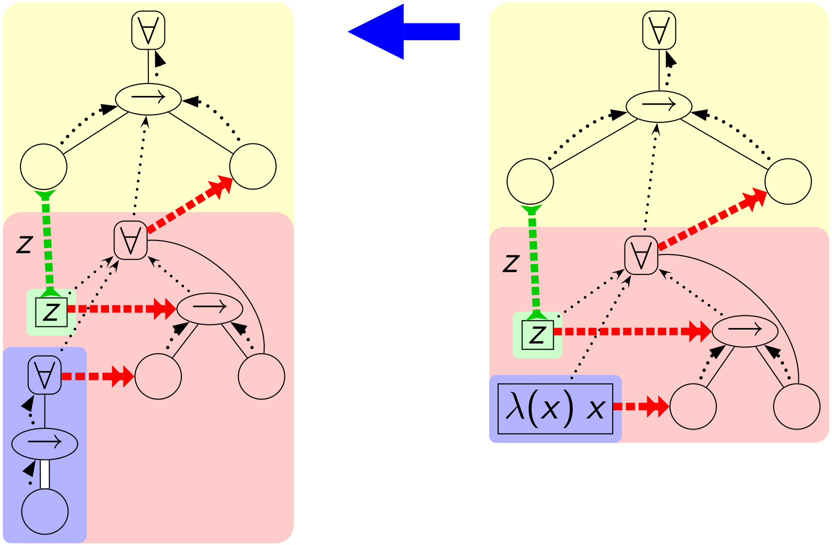

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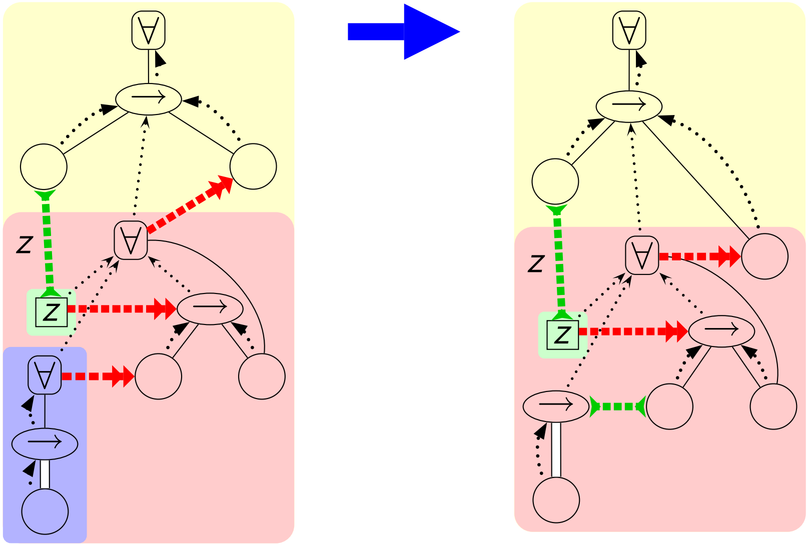

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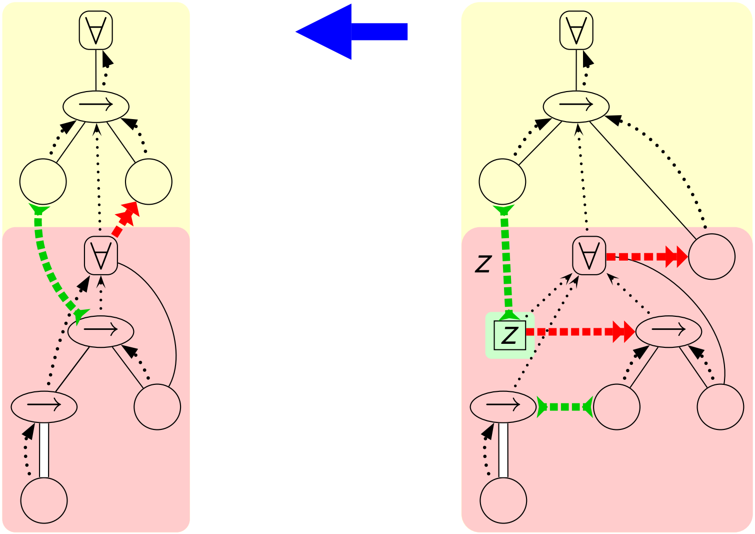

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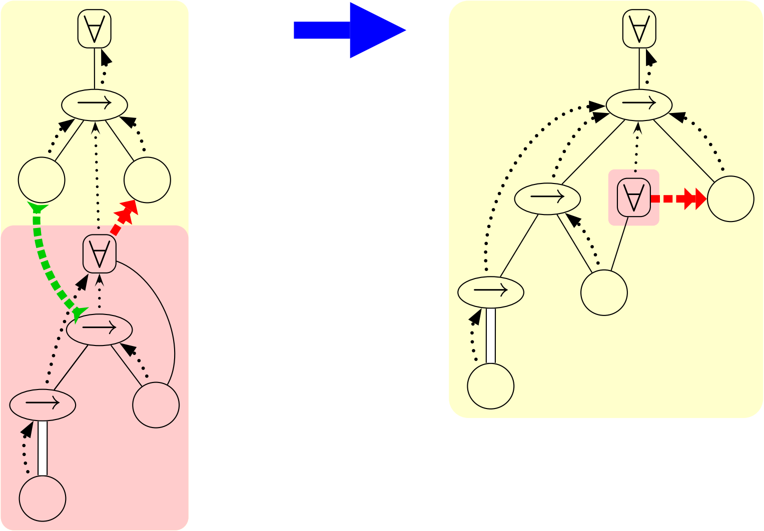

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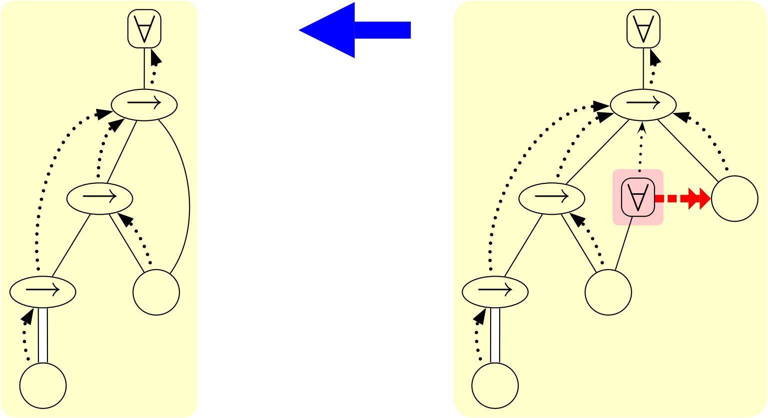

Type inference with typing constraints (demo)

skip back

$$\boxed{\lambda(z)\ \ (z : \sigma_{\mathsf{id}})}$$

Type inference with typing constraints (demo)

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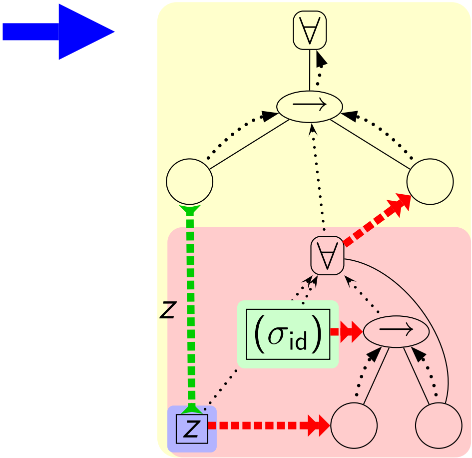

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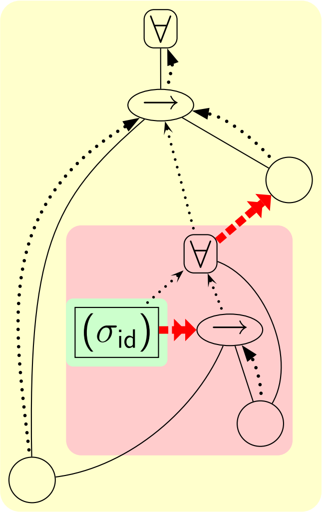

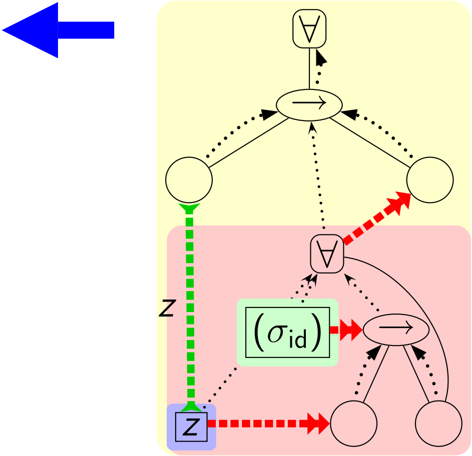

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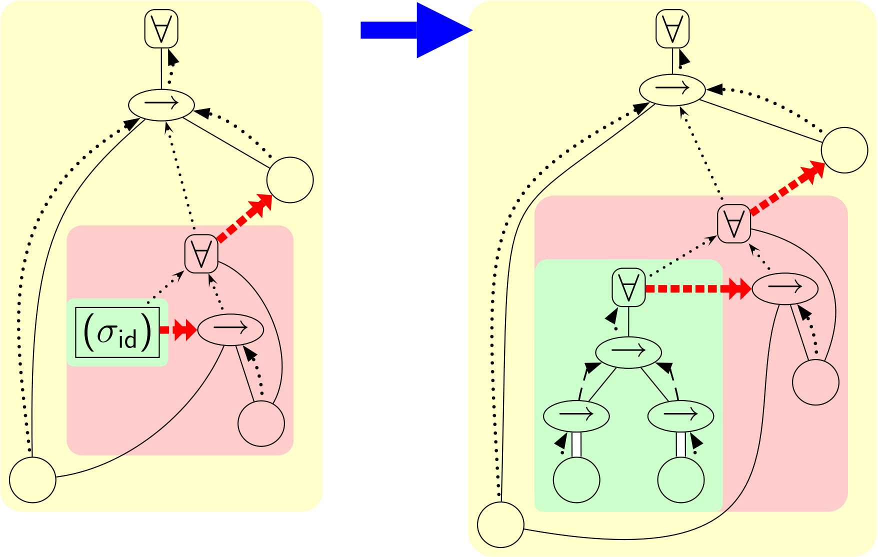

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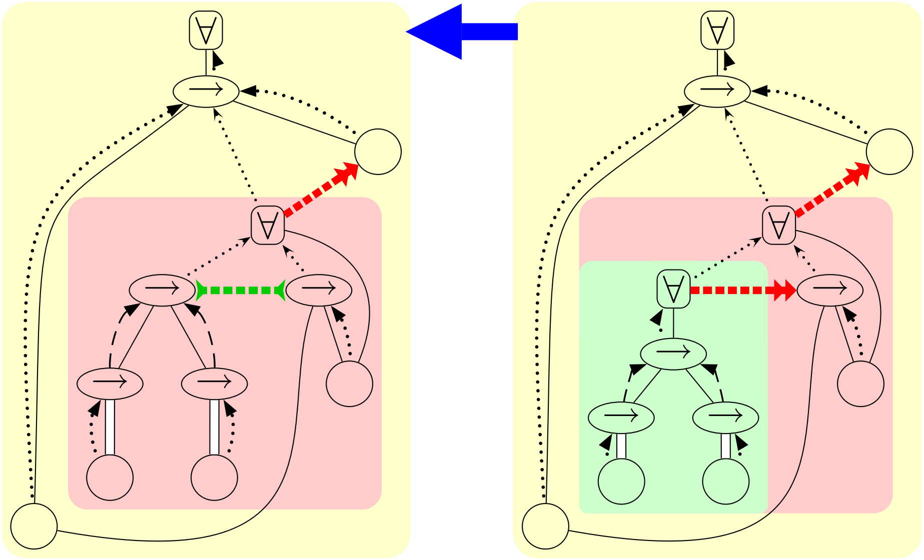

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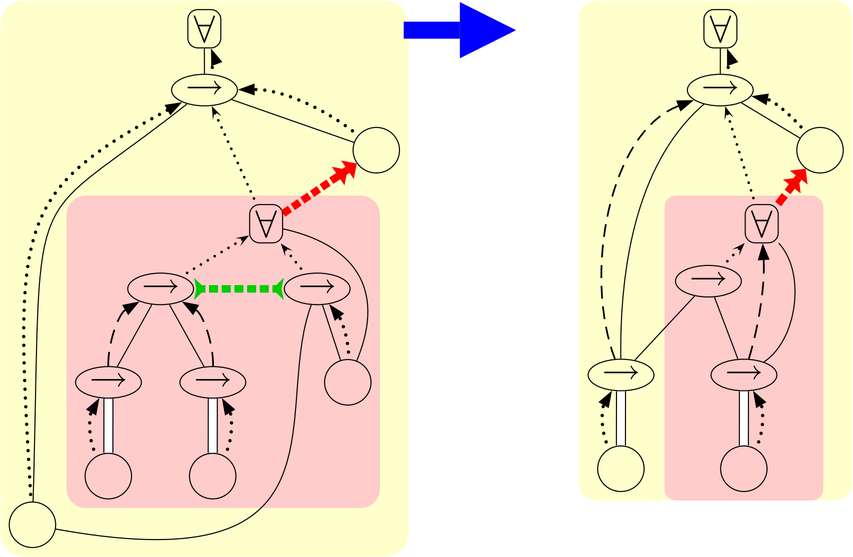

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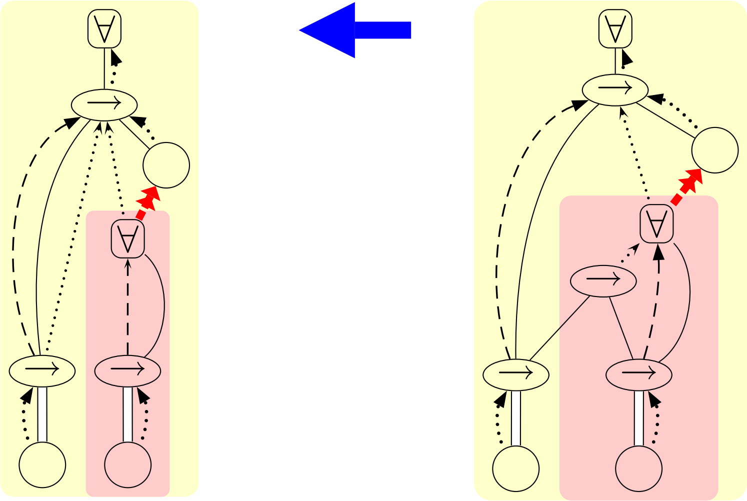

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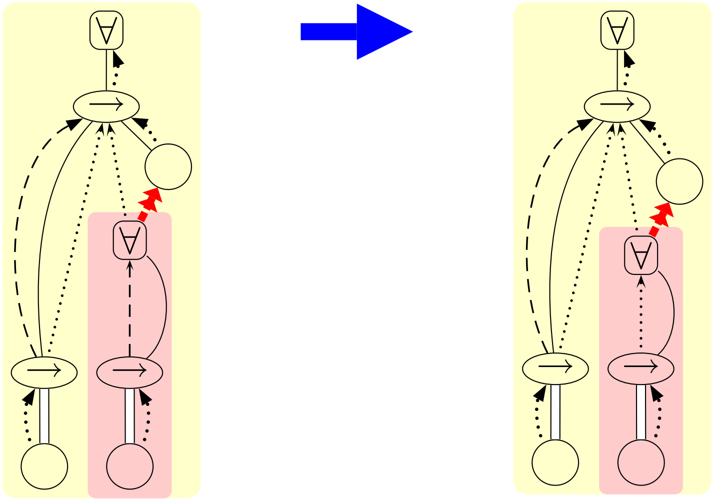

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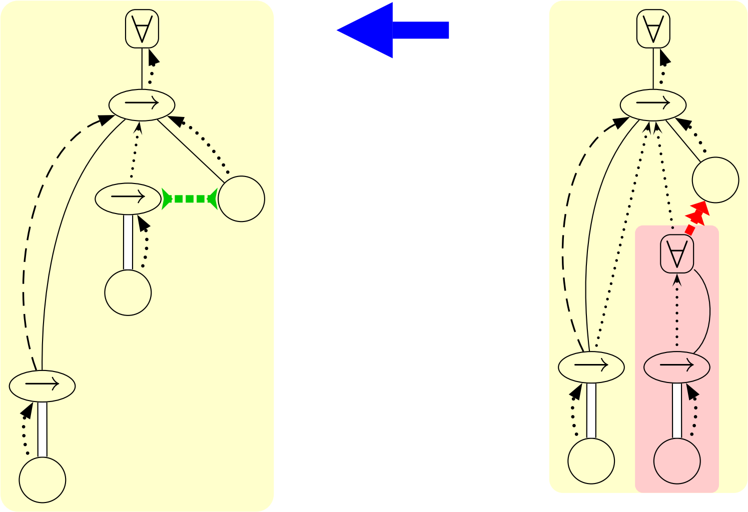

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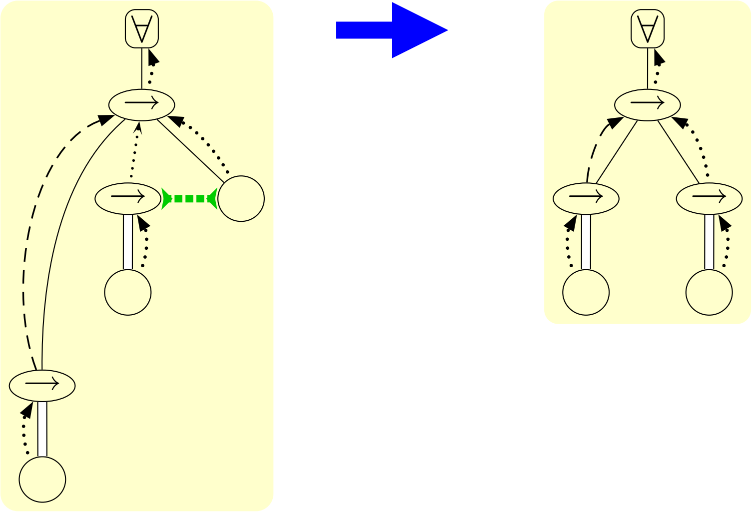

◀ back

More examples

Encoding of existential types, *e.g.* $\exists\beta.\beta \times \beta \to \alpha$

type α func $= \forall(\gamma)\ \forall\ (\delta = \forall(\beta)\ \beta * (\beta \to \alpha) \to \gamma)\ \delta \to \gamma$

val pack z = fun (f : $\exists(\gamma)\ \forall(\beta)\ \beta * (\beta \to \alpha) \to \gamma) \to$ f z;;

val pack : $\forall(\alpha)\ \forall(\beta)\ \alpha * (\alpha \to \beta) \to (\forall(\gamma)\ (\forall(\delta)\ \delta * (\delta \to \beta) \to \gamma) \to \gamma)$

let packed_int = pack (1, fun x $\to$ x+1);;
let packed_pair = pack (1, fun x $\to$ (x, x));;

let v = packed_int (fun p $\to$ (snd p) (fst p));;

About Rigid ML^F

back

Rigid ML^F lies very close to ML^F

- It uses and relies on (Shallow) ML^F internally.
- It projects ML^F principal types into System-F types at let-bindings, by raising variable bindings as much as possible.

Rigid ML^F looses important properties of ML^F

- There are no principal types *per se*.
  - Rigid ML^F pretends to have principal types, but this is in an ad hoc manner, using a non logical typing rule for Let-bindings with a premise that blocks free uses of type-instantiation.
- let $x = \lambda(z : \sigma)\ z$ in a_2 may be accepted while let $x = \lambda(z)\ z$ in a_2 would be rejected.
- Rigid ML^F is not invariant by let-expansion (which signs the lost of truly principal types).

About Rigid ML^{F}

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Rigid ML^{F} looses important properties of ML^{F}

- There are no principal types *per se*.
- Rigid ML^{F} is not invariant by let-expansion (which signs the lost of truly principal types).

Rigid ML^{F} is a subset of System F

- This is both its interest and its problem.

What would be an intermediate language for ML^F?

Problem

- Subject reduction is only proved in $i\mathrm{ML}^F$, which has the same type erasure as $e\mathrm{ML}^F$.
  - This ensures correctness of $i\mathrm{ML}^F$
  - But does not help to propagate annotations during reduction (or other program transformations)
- Even so, $e\mathrm{ML}^F$ requires type inference, which is not a local process.

Solution

- Introduce a fully explicit version of $x\mathrm{ML}^F$ (easy)
- Instrument reduction rules to keep track of types during reduction (not entirely trivial)
- This has to be investigated.

Sharing of abstract nodes is irreversible (implicitly)

back

Can you show an example illustrating the difference?

Fact: $\forall(\alpha \Rightarrow \sigma)\ \alpha \to \alpha \not\leq \forall(\alpha \Rightarrow \sigma, \alpha' \Rightarrow \sigma)\ \alpha \to \alpha'$

Observe that:

- $\lambda(z)\ z : \forall(\alpha \Rightarrow \sigma)\ \alpha \to \alpha$
- $(\_ : \sigma) : \forall(\alpha \Rightarrow \sigma, \alpha' \Rightarrow \sigma)\ \alpha \to \alpha'$

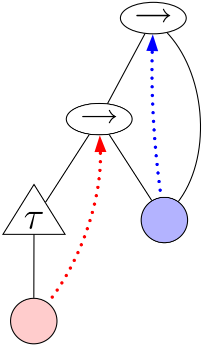
Then, the context $a \triangleq \lambda(x)\ []\ x\ x$ distinguishes those two expressions.

- $a[\lambda(z)\ z]$ is ill-typed.
  (As it uses no type annotation and it is ill-typed in ML)
- $a[(\_ : \sigma)]$ is well-typed.

System-F types (encoding of existential types)

back

$$\forall(\alpha)\ (\forall(\beta)\ \tau_\beta \to \alpha) \to \alpha$$



System-F types (encoding of existential types)

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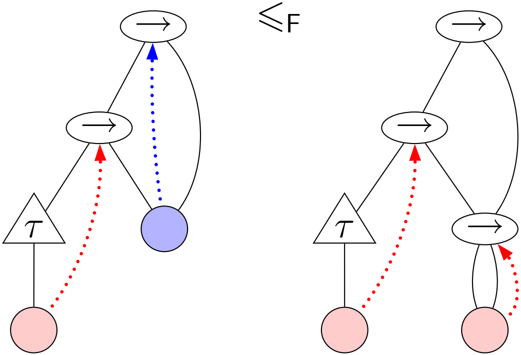

$$(\forall(\beta)\ \tau_\beta \to \forall(\alpha)\ \alpha \to \alpha) \to \forall(\alpha)\ \alpha \to \alpha$$

System-F types (encoding of existential types)

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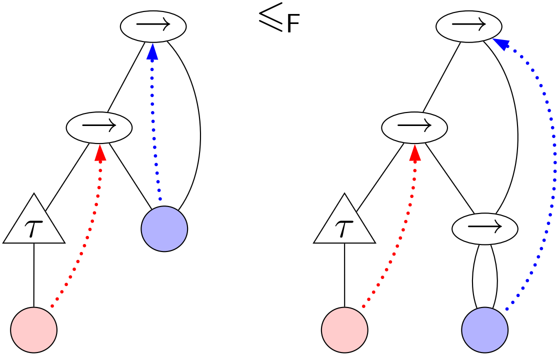
$$\forall(\alpha)\ (\forall(\beta)\ \tau_\beta \to \alpha) \to \alpha$$



$$\forall(\alpha)\ (\forall(\beta)\ \tau_\beta \to \alpha \to \alpha) \to \alpha \to \alpha$$

◀ back

Type annotations

back

$$\frac{\alpha \Rightarrow \sigma, \beta \Rightarrow \sigma \vdash \sigma \leqslant \alpha \text{ and } \sigma \leqslant \beta}{\alpha \Rightarrow \sigma, \beta \Rightarrow \sigma \vdash \begin{array}{ll} & \forall(\alpha' \Rightarrow \sigma)\,\forall(\beta' \Rightarrow \sigma)\,\alpha' \to \beta' \\ \leqslant & \forall(\alpha' \Rightarrow \alpha)\,\forall(\beta' \Rightarrow \beta)\,\alpha' \to \beta' \\ & \diamond \\ & \alpha \to \beta \end{array}}$$

$$\dfrac{\dfrac{\dfrac{\alpha \Rightarrow \sigma, x : \alpha, \beta \Rightarrow \sigma \vdash (\_ : \sigma) : \alpha \to \beta \qquad \alpha \Rightarrow \sigma, x : \alpha, \beta \Rightarrow \sigma \vdash x : \alpha}{\alpha \Rightarrow \sigma, x : \alpha, \beta \Rightarrow \sigma \vdash (x : \sigma) : \beta}}{\alpha \Rightarrow \sigma, x : \alpha \vdash (x : \sigma) : \forall(\beta \Rightarrow \sigma)\,\beta}}{\boxed{\alpha \Rightarrow \sigma, x : \alpha \vdash (x : \sigma) : \sigma}}$$

Type annotations

back

$$
\dfrac{\dfrac{\dfrac{\alpha \Rightarrow \sigma_{\text{id}}, x : \alpha \vdash (x : \sigma_{\text{id}}) : \sigma_{\text{id}}}{\alpha \Rightarrow \sigma_{\text{id}}, x : \alpha \vdash (x : \sigma_{\text{id}}) : \alpha \to \alpha} \qquad \alpha \Rightarrow \sigma_{\text{id}}, x : \alpha \vdash x : \alpha}{\dfrac{\alpha \Rightarrow \sigma_{\text{id}}, x : \alpha \vdash (x : \sigma_{\text{id}})\ x : \alpha}{\alpha \Rightarrow \sigma_{\text{id}} \vdash \lambda(x)\ (x : \sigma_{\text{id}})\ x : \alpha \to \alpha}}}{\vdash \lambda(x)\ (x : \sigma_{\text{id}})\ x : \forall(\alpha \Rightarrow \sigma_{\text{id}})\ \alpha \to \alpha}
$$